Statistics Canada Statistique Canada

Canada

Statistics Canada
www.statcan.gc.ca

# Choice of Index Number Formula and the Upper-level Substitution Bias in the Canadian CPI

**Ning Huang, Waruna Wimalaratne and Brent Pollard**

14th Ottawa Group Meeting

May 20, 2015

# Outline

- Motivation
- Data Sources and Construction
- Official CPI—the Lowe Index
- Symmetrically Weighted Price Indices
- Asymmetrically Weighted Price Indices
- Other Weighted Price Indices
- Conclusions

# Motivation

- Inherent limitation of the Lowe index number formula
  - The Consumer Prices Division (CPD) at Statistics Canada currently uses the Lowe index formula for the upper-level aggregation of its CPI
  - It cannot account for consumers' price-induced substitution behaviour due to the fixed-basket concept

  => Upper-level commodity substitution bias

# Motivation (Cont.)

- Statistics Canada's action in 2013
  - Moved to a more frequent basket-update schedule
  - Implemented the 2011 basket with a shorter implementation lag

- Approaches for reducing the substitution bias
  - Update the basket more frequently;
  - Implement new baskets in a more timely fashion;
  - Apply alternative index number formulas.

# Motivation (Cont.)

- Existing literature about reducing the substitution bias through adopting alternative index number formulas

  - Hansen (2007) : Lowe index vs. Young index
  - Lent and Dorfman (2009): weighted arithmetic average of base-period indices (e.g. Laspeyres + Geometric Laspeyres)
  - Armknecht and Silver (2012): simple geometric average of the arithmetic mean and geometric mean (e.g. Lowe + Geometric Young)

# Motivation (Cont.)

- Investigate the impacts of applying different index number formulas on reducing the upper-level commodity-substitution bias in the Canadian CPI under the current economic situation.

# Data Sources and Construction

- Data sources

  - The Survey of Household Spending (SHS)

  - Unlinked price indices at basic class level for Canada

[1]The base year expenditure on category i is $p_i^b q_i^b$. This expenditure weight is "updated" into the month 0 hybrid expenditure weight $w_i$ by multiplying $p_i^b q_i^b$ by the category i ratio of the month 0 price $p_i^0$ divided by the corresponding category i base year category i price $p_i^b$; i.e., $w_i \equiv p_i^b q_i^b (p_i^0/p_i^b)$.

# Data sources and construction

- Data Construction
  - To maintain uniformity and to avoid complications, the 2005 classification was used
  - Basket updates occurred in 2001, 2005, 2009, and 2011, thus the official baskets were used for these four years.
  - Expenditure weights at the basic class level were constructed for the following years: 2000, 2002, 2003, 2004, 2006, 2007, 2008, and 2010.

[1]The base year expenditure on category i is $p_i^b q_i^b$. This expenditure weight is "updated" into the month 0 hybrid expenditure weight $w_i$ by multiplying $p_i^b q_i^b$ by the category i ratio of the month 0 price $p_i^0$ divided by the corresponding category i base year category i price $p_i^b$; i.e., $w_i \equiv p_i^b q_i^b (p_i^0/p_i^b)$.

# Data sources and construction

- Data Construction
  - Weight calculation
    - Where the SHS lacked detail for some basic classes, the procedure of backward and forward price updating was used. This affected approximately 30% of the basket, mainly the food component.
    - For example:

$$p_i^{2004} q_i^{2004} \equiv \left(5/8\right) v_i^{2001} \frac{p_i^{2004}}{p_i^{2001}} + \left(3/8\right) v_i^{2009} \frac{p_i^{2004}}{p_i^{2009}}$$

Where:

$$v_i^{2001} = p_i^{2001} \times q_i^{2001} \qquad v_i^{2009} = p_i^{2009} \times q_i^{2009}$$

# Calculation of Price Relatives

- Index calculation

  - Using existing data from the official CPI at the national level, for the thirteen years from 2000 to 2012, price relatives were calculated for each basic class based to January 2000 = 100.

  - Special calculations were required for some classes due to changes in aggregation, addition of new classes and complications arising from "*not elsewhere specified*" (NES) categories.

# Official CPI—the Lowe Index

- The Canadian CPI is published monthly on predetermined release dates
- Two-stage aggregation is used in the CPI calculation
- Lowe index number is used for the upper-level aggregation

# Official CPI—the Lowe Index

The direct Lowe index can be constructed as follows:

$$P_{Lo}(p^0, p^t, q^b) = \frac{\sum_{i=1}^{N} p_i^t \times q_i^b}{\sum_{i=1}^{N} p_i^0 \times q_i^b} = \sum_{i=1}^{N} \left( \frac{p_i^t}{p_i^0} \right) \times \frac{p_i^0 \times q_i^b}{\sum_{i=1}^{N} p_i^0 \times q_i^b} = \sum_{i=1}^{N} \left( \frac{p_i^t}{p_i^0} \right) s_i^{0:b}$$

where the hybrid expenditures are obtained through a price-updating process:

$$p_i^0 \times q_i^b = \left( \frac{p_i^0}{p_i^b} \right) \left( p_i^b \times q_i^b \right)$$

# Symmetrically Weighted Price Indices

Treat prices and quantities in both periods being compared symmetrically so that the indices account for changes in expenditure patterns over the two price comparison periods.

- Superlative indices:

  Fisher, Walsh, and Törnqvist indices

  - Approximate each other closely
  - Approximate the underlying conditional cost-of-living index based on a flexible functional form
  - Recommended as the target index for the upper-level index

# Symmetrically Weighted Price Indices

- Superlative indices:

  Fisher, Walsh, and Törnqvist indices

  Fisher index:

$$P_F^{t/0} = \left( \frac{\sum_{i=1}^{N} p_i^t \times q_i^0}{\sum_{i=1}^{N} p_i^0 \times q_i^0} \times \frac{\sum_{i=1}^{N} p_i^t \times q_i^t}{\sum_{i=1}^{N} p_i^0 \times q_i^t} \right)^{1/2} = \left( P_L^{t/0} \times P_P^{t/0} \right)^{1/2}$$

# Symmetrically Weighted Price Indices

- Superlative indices:

  Fisher, Walsh, and Törnqvist indices

Walsh index:

$$P_W^{t/0} = \frac{\sum_{i=1}^{N} p_i^t \times \sqrt{q_i^t q_i^0}}{\sum_{i=1}^{N} p_i^0 \times \sqrt{q_i^t q_i^0}}$$

Törnqvist index:

$$P_T^{t/0} = \prod_{i=1}^{N} \left( \frac{p_i^t}{p_i^0} \right)^{\frac{1}{2}\left(s_i^0 + s_i^t\right)}$$

# Symmetrically Weighted Price Indices

- Superlative indexes
- Other Symmetrically Weighted Price Indices

  Marshall-Edgeworth, Drobisch and Un-named price indices

$$P_{ME}^{t/0} = \frac{\sum_{i=1}^{N} p_i^t \times \left[ \left( q_i^0 + q_i^t \right) / 2 \right]}{\sum_{i=1}^{N} p_i^0 \times \left[ \left( q_i^0 + q_i^t \right) / 2 \right]}$$

$$P_{UN}^{t/0} = \sum_{i=1}^{N} \frac{1}{2} \left( s_i^0 + s_i^t \right) \left( \frac{p_i^t}{p_i^0} \right)$$

$$P_{DR}^{t/0} = \frac{1}{2} \left( P_L^{t/0} + P_P^{t/0} \right) = \frac{1}{2} \left( \frac{\sum_{i=1}^{N} p_i^t \times q_i^0}{\sum_{i=1}^{N} p_i^0 \times q_i^0} + \frac{\sum_{i=1}^{N} p_i^t \times q_i^t}{\sum_{i=1}^{N} p_i^0 \times q_i^t} \right)$$

# Symmetrically Weighted Price Indices

- The chain-linked symmetrically weighted price indices (2003=100)

| Year (t) | Superlative Indices | | | Other Symmetrically Weighted Indices | | |
|---|---|---|---|---|---|---|
| | Fisher | Walsh | Törnqvist | ME | Drobisch | Un-named |
| 2003 | 100.000 | 100.000 | 100.000 | 100.000 | 100.000 | 100.000 |
| 2004 | 101.728 | 101.730 | 101.730 | 101.727 | 101.728 | 101.806 |
| 2005 | 103.746 | 103.750 | 103.750 | 103.744 | 103.746 | 103.919 |
| 2006 | 105.475 | 105.480 | 105.482 | 105.472 | 105.475 | 105.718 |
| 2007 | 107.401 | 107.409 | 107.410 | 107.398 | 107.401 | 107.716 |
| 2008 | 109.624 | 109.632 | 109.633 | 109.619 | 109.624 | 110.102 |
| 2009 | 109.670 | 109.684 | 109.688 | 109.663 | 109.670 | 110.393 |
| 2010 | 111.404 | 111.422 | 111.422 | 111.397 | 111.405 | 112.234 |
| 2011 | 114.389 | 114.408 | 114.405 | 114.381 | 114.390 | 115.374 |

# Symmetrically Weighted Price Indices

- Superlative indices
- Several other symmetrically weighted price indices
- Target index in this study
  - Chained Fisher index with annual basket-updating

$$P_{Ch\_F}^{2011/2002} = P_F^{2003/2002} \times P_F^{2004/2003} \times P_F^{2005/2004} \times \ldots \times P_F^{2010/2009} \times P_F^{2011/2010}$$

$$= \left( P_L^{2003/2002} \times P_P^{2003/2002} \right)^{1/2} \times P_F^{2004/2003} \times P_F^{2005/2004} \times \ldots \times P_F^{2010/2009} \times P_F^{2011/2010}$$

# Symmetrically Weighted Price Indices

- The difference between the chained Fisher and other symmetric price indices

| Year (t) | Superlative Indices | | | Other Symmetrically Weighted Indices | | |
|---|---|---|---|---|---|---|
| | Fisher | Walsh | Törnqvist | ME | Drobisch | Un-named |
| 2003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2004 | 0.000 | 0.001 | 0.001 | -0.001 | 0.000 | 0.078 |
| 2005 | 0.000 | 0.004 | 0.004 | -0.002 | 0.000 | 0.173 |
| 2006 | 0.000 | 0.005 | 0.007 | -0.004 | 0.000 | 0.243 |
| 2007 | 0.000 | 0.008 | 0.009 | -0.004 | 0.000 | 0.315 |
| 2008 | 0.000 | 0.008 | 0.009 | -0.005 | 0.000 | 0.478 |
| 2009 | 0.000 | 0.014 | 0.018 | -0.007 | 0.000 | 0.723 |
| 2010 | 0.000 | 0.018 | 0.018 | -0.008 | 0.000 | 0.830 |
| 2011 | 0.000 | 0.019 | 0.016 | -0.008 | 0.000 | 0.984 |

# Asymmetrically Weighted Price Indices

Weights used to aggregate the elementary price indices are not associated with both price comparison periods.

- Laspeyres price index: base-period arithmetic index

$$P_L^{t/0} = \frac{\sum_{i=1}^{N} p_i^t q_i^0}{\sum_{i=1}^{N} p_i^0 q_i^0} = \sum_{i=1}^{N} s_i^0 \left( \frac{p_i^t}{p_i^0} \right)$$

# Asymmetrically Weighted Price Indices

- Laspeyres index: base-period arithmetic index

  Decomposition of the difference in the index value between the Laspeyres index and the Lowe index:

$$P_L^{t/0} - P_{Lo}^{t/0} = \frac{\sum_{i=1}^{N} p_i^t q_i^0}{\sum_{i=1}^{N} p_i^0 q_i^0} - \frac{\sum_{i=1}^{N} p_i^t q_i^b}{\sum_{i=1}^{N} p_i^0 q_i^b}$$

$$= \sum_{i=1}^{N} \left( \frac{p_i^t}{p_i^0} - P_L^{t/0} \right) \left( \frac{q_i^0}{q_i^b} - Q_P^{0/b} \right) \frac{s_i^{0:b}}{Q_P^{0/b}}$$

# Asymmetrically Weighted Price Indices

Weights used to aggregate the elementary price indices are not associated with both price comparison periods.

- Paasche price index: current-period arithmetic index

$$P_P^{t/0} = \frac{\sum_{i=1}^{N} p_i^t q_i^t}{\sum_{i=1}^{N} p_i^0 q_i^t} = \left[ \sum_{i=1}^{N} s_i^t \left( \frac{p_i^t}{p_i^0} \right)^{-1} \right]^{-1}$$

# Asymmetrically Weighted Price Indices

- Paasche index: current-period arithmetic index

  Decomposition of the difference in the index value between the Paasche index and the Lowe index:

$$P_P^{t/0} - P_{Lo}^{t/0} = \frac{\sum_{i=1}^{N} p_i^t q_i^t}{\sum_{i=1}^{N} p_i^0 q_i^t} - \frac{\sum_{i=1}^{N} p_i^t q_i^b}{\sum_{i=1}^{N} p_i^0 q_i^b}$$

$$= \sum_{i=1}^{N} \left( \frac{p_i^t}{p_i^0} - P_L^{t/0} \right) \left( \frac{q_i^t}{q_i^b} - Q_{Lo}^{t/b} \right) \frac{s_i^{0:b}}{Q_{Lo}^{0/b}}$$

# Asymmetrically Weighted Price Indices

Weights used to aggregate the elementary price indices are not associated with both price comparison periods.

- Palgrave price index: current-period arithmetic index

$$P_{Pal}^{t/0} = \sum_{i=1}^{N} s_i^t \left( \frac{p_i^t}{p_i^0} \right)$$

# Asymmetrically Weighted Price Indices

- Palgrave index: current-period arithmetic index

  Decomposition of the difference in the index value between the Palgrave index and the Lowe index:

$$P_{Pal}^{t/0} - P_{Lo}^{t/0} = \sum_{i=1}^{N} s_i^t \left( \frac{p_i^t}{p_i^0} \right) - \frac{\sum_{i=1}^{N} p_i^t q_i^b}{\sum_{i=1}^{N} p_i^0 q_i^b}$$

$$= \sum_{i=1}^{N} \left( \frac{p_i^t}{p_i^0} - P_{Lo}^{t/0} \right) \left( s_i^t - s_i^{0:b} \right)$$

# Asymmetrically Weighted Price Indices

- Palgrave index: current-period arithmetic index

  Decomposition of the difference in the index value between the Palgrave index and the Laspeyres index:

$$P_{Pal}^{t/0} - P_L^{t/0} = \sum_{i=1}^{N} s_i^t \left( \frac{p_i^t}{p_i^0} \right) - \frac{\sum_{i=1}^{N} p_i^t q_i^0}{\sum_{i=1}^{N} p_i^0 q_i^0}$$

$$= \sum_{i=1}^{N} \left( \frac{p_i^t}{p_i^0} - P_L^{t/0} \right) \left( s_i^t - s_i^0 \right)$$

# Asymmetrically Weighted Price Indices

- The chained asymmetrically weighted price indices:

| | Target Index | Official Index | Asymmetrically Weighted Price Indices | | |
|---|---|---|---|---|---|
| | Fisher | Lowe | Laspeyres | Paasche | Palgrave |
| 2003 | 100.000 | 100.000 | 100.000 | 100.000 | 100.000 |
| 2004 | 101.743 | 101.916 | 101.802 | 101.684 | 101.840 |
| 2005 | 103.768 | 104.176 | 103.925 | 103.611 | 103.955 |
| 2006 | 105.503 | 106.204 | 105.712 | 105.295 | 105.777 |
| 2007 | 107.445 | 108.310 | 107.687 | 107.202 | 107.826 |
| 2008 | 109.694 | 110.813 | 110.068 | 109.320 | 110.267 |
| 2009 | 109.753 | 111.095 | 110.343 | 109.166 | 110.601 |
| 2010 | 111.503 | 112.921 | 112.140 | 110.870 | 112.510 |
| 2011 | 114.466 | 115.987 | 115.123 | 113.813 | 115.759 |

# Asymmetrically Weighted Price Indices

- The chained asymmetrically weighted price indices:

| | Target Index | Official Index | Asymmetrically Weighted Price Indices | | |
|---|---|---|---|---|---|
| | Fisher | Lowe | Laspeyres | Paasche | Palgrave |
| 2004 | 1.743 | 1.916 | 1.802 | 1.684 | 1.840 |
| 2005 | 1.990 | 2.217 | 2.086 | 1.895 | 2.077 |
| 2006 | 1.672 | 1.947 | 1.719 | 1.625 | 1.752 |
| 2007 | 1.840 | 1.983 | 1.869 | 1.811 | 1.937 |
| 2008 | 2.093 | 2.311 | 2.211 | 1.975 | 2.264 |
| 2009 | 0.054 | 0.254 | 0.250 | -0.141 | 0.303 |
| 2010 | 1.595 | 1.644 | 1.628 | 1.561 | 1.726 |
| 2011 | 2.657 | 2.715 | 2.660 | 2.655 | 2.888 |
| Average Growth Rate (2003-2011) | 1.703 | 1.871 | 1.776 | 1.630 | 1.846 |

# Other Weighted Price Indices

Price indices constructed with the same data required to compile the Lowe index:

- Young index

$$P_Y(p^0, p^t, s^b) = \sum_i s_i^b \left( \frac{p_i^t}{p_i^0} \right) = \sum_i \left( \frac{p_i^t}{p_i^0} \right) \times \frac{p_i^b q_i^b}{\sum_i p_i^b q_i^b}$$

# Other Weighted Price Indices

- Decomposition of the difference between the Lowe index and the Young index:

$$P_{Lo}(p^0, p^t, s^{0:b}) - P_Y(p^0, p^t, s^b) = \sum_i \frac{p_i^t}{p_i^0}(s_i^{0:b} - s_i^b)$$

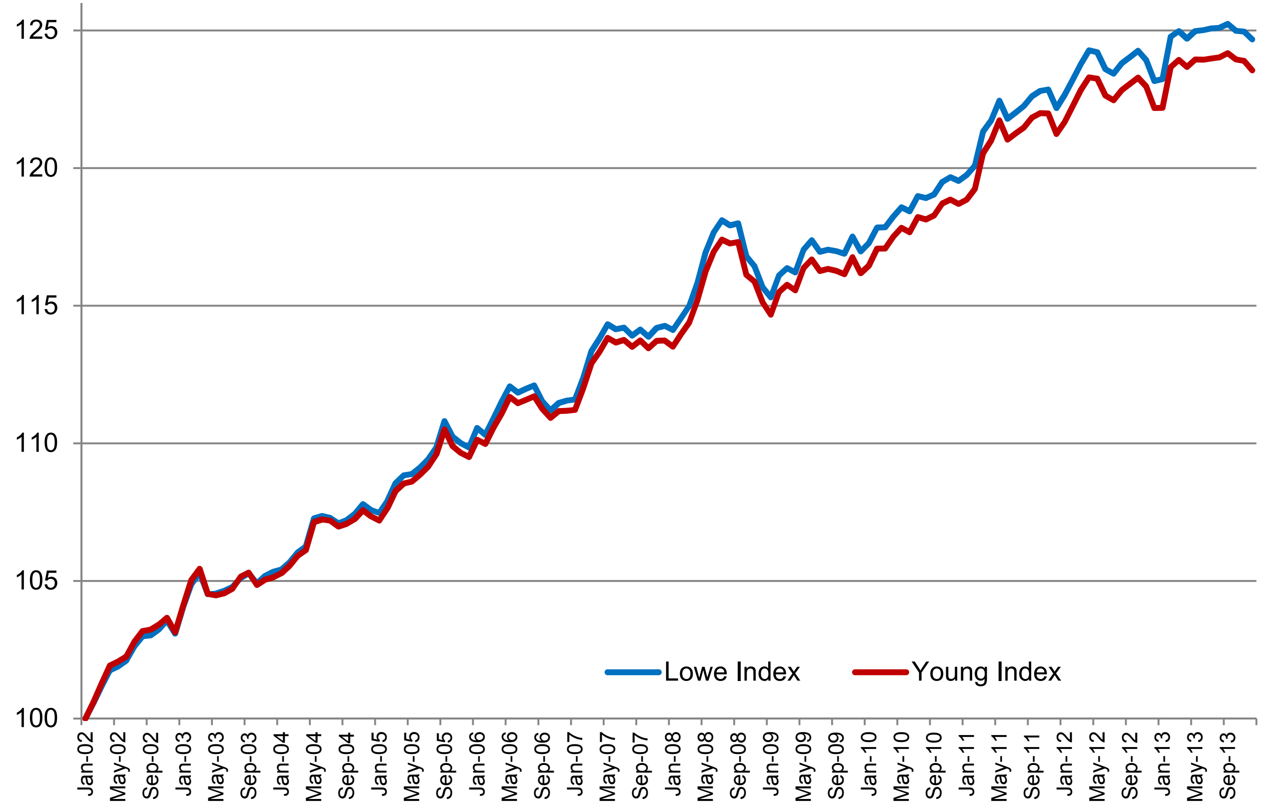
$$= \sum_i \left[ \frac{p_i^t}{p_i^0} - P_Y^t(p^0, p^t, s^b) \right] \times \left[ \frac{p_i^0}{p_i^b} - P_Y^0(p^b, p^0, s^b) \right] \left( \frac{s_i^b}{P_Y^0(p^b, p^0, s^b)} \right)$$

Long-term consistent price trends (unidirectional)

=> Lowe index > Young index

# Other Weighted Price Indices

- Lowe index vs. Young index



# Other Weighted Price Indices

- The differences between the Lowe index and Young index

| | Chained Lowe index – Chained Young index | | |
|---|---|---|---|
| | Baskets updated every 2 years, lag of 13 months | Baskets updated every year, lag of 13 months | Baskets updated every year, lag of 12 months |
| 2003 | 0.015 | -0.028 | -0.049 |
| 2004 | 0.147 | 0.145 | 0.062 |
| 2005 | 0.293 | 0.318 | 0.147 |
| 2006 | 0.358 | 0.378 | 0.298 |
| 2007 | 0.444 | 0.357 | 0.350 |
| 2008 | 0.638 | 0.510 | 0.521 |
| 2009 | 0.687 | 0.291 | 0.158 |
| 2010 | 0.778 | 0.236 | 0.057 |
| 2011 | 0.805 | 0.370 | 0.200 |
| 2012 | 0.969 | 0.511 | 0.313 |
| 2013 | 1.066 | 0.533 | 0.319 |

# Other Weighted Price Indices

Price indices constructed with the same data required to compile the Lowe index

- Geometric Price Indices
  - Geometric Lowe Index:

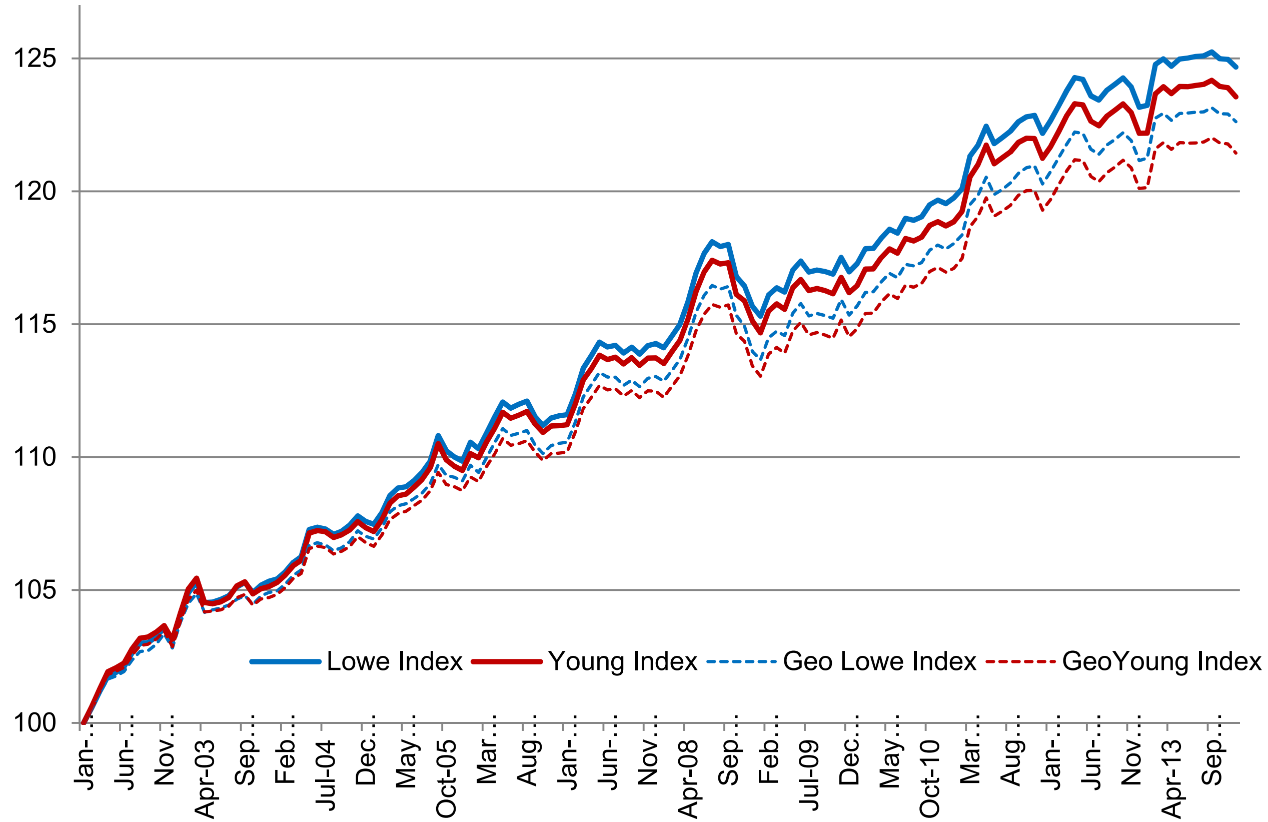
$$P_{GLo}(p^0, p^t, s^{0:b}) = \prod_{i=1}^{N} \left( \frac{p_i^t}{p_i^0} \right)^{s_i^{0:b}}$$

  - Geometric Young Index:

$$P_{GY}(p^0, p^t, s^b) = \prod_{i=1}^{N} \left( \frac{p_i^t}{p_i^0} \right)^{s_i^b}$$

# Other Weighted Price Indices

- Geometric weighted and arithmetic weighted indices



# Other Weighted Price Indices

Price indices constructed with the same data required to compile the Lowe index

- Lloyd-Moulton index and its modified version
  - Lloyd-Moulton index:

$$P_{LM}^{t/0} = \left\{ \sum_{i=1}^{N} s_i^0 \left( \frac{p_i^t}{p_i^0} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}$$

  - Modified Lloyd-Moulton:

$$P_{ModLM}^{t/0} = \left\{ \sum_{i=1}^{N} s_i^b \left( \frac{p_i^t}{p_i^0} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}$$

# Other Weighted Price Indices

Price indices constructed with the same data required to compile the Lowe index

- Arithmetic AG Mean (LD) index and its modified version
  - LD index:

$$P_{LD}(p^0, p^t, s^0) = \sigma \prod_{i=1} \left( \frac{p_i^t}{p_i^0} \right)^{s_i^0} + (1-\sigma) \sum_{i=1} \left( \frac{p_i^t}{p_i^0} \right) s_i^0$$

  - Modified LD index:

$$P_{ModLD}(p^0, p^t, s^0) = \sigma \prod_{i=1} \left( \frac{p_i^t}{p_i^0} \right)^{s_i^b} + (1-\sigma) \sum_{i=1} \left( \frac{p_i^t}{p_i^0} \right) s_i^b$$

# Other Weighted Price Indices

Price indices constructed with the same data required to compile the Lowe index

- Lloyd-Moulton index and its modified version
- Arithmetic AG Mean (LD) index and its modified version

Estimates of the elasticity of substitution between commodities:

| | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | Aver-age |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| σ-F | 0.495 | 0.740 | 0.444 | 0.727 | 1.023 | 0.862 | 0.538 | 0.840 | 0.882 | 0.409 | 0.350 | 0.665 |
| σ-W | 0.506 | 0.729 | 0.448 | 0.709 | 0.997 | 0.844 | 0.499 | 0.837 | 0.859 | 0.370 | 0.347 | 0.650 |
| σ-T | 0.510 | 0.727 | 0.455 | 0.709 | 0.992 | 0.821 | 0.510 | 0.841 | 0.845 | 0.415 | 0.367 | 0.654 |

# Other Weighted Price Indices

The Modified LM index corresponding to different estimates of elasticity of substitution (σ)

| σ | Index Values (2003-2011) | Annual Inflation Rate | Difference in Annual Inflation Rate |
|---|---|---|---|
| Fisher | 114.389 | 1.695 | 0.000 |
| 0.347 | 114.981 | 1.760 | 0.066 |
| 0.409 | 114.891 | 1.750 | 0.056 |
| 0.499 | 114.758 | 1.736 | 0.041 |
| 0.538 | 114.700 | 1.729 | 0.035 |
| 0.654 | 114.530 | 1.710 | 0.016 |
| 0.665 | 114.514 | 1.709 | 0.014 |
| 0.740 | 114.402 | 1.696 | 0.001 |
| 0.859 | 114.226 | 1.677 | -0.018 |
| 1.023 | 113.983 | 1.649 | -0.045 |

# Other Weighted Price Indices

Price indices constructed with the same data required to compile the Lowe index

- Geometric mean of the weighted arithmetic average and weighted geometric average, including:

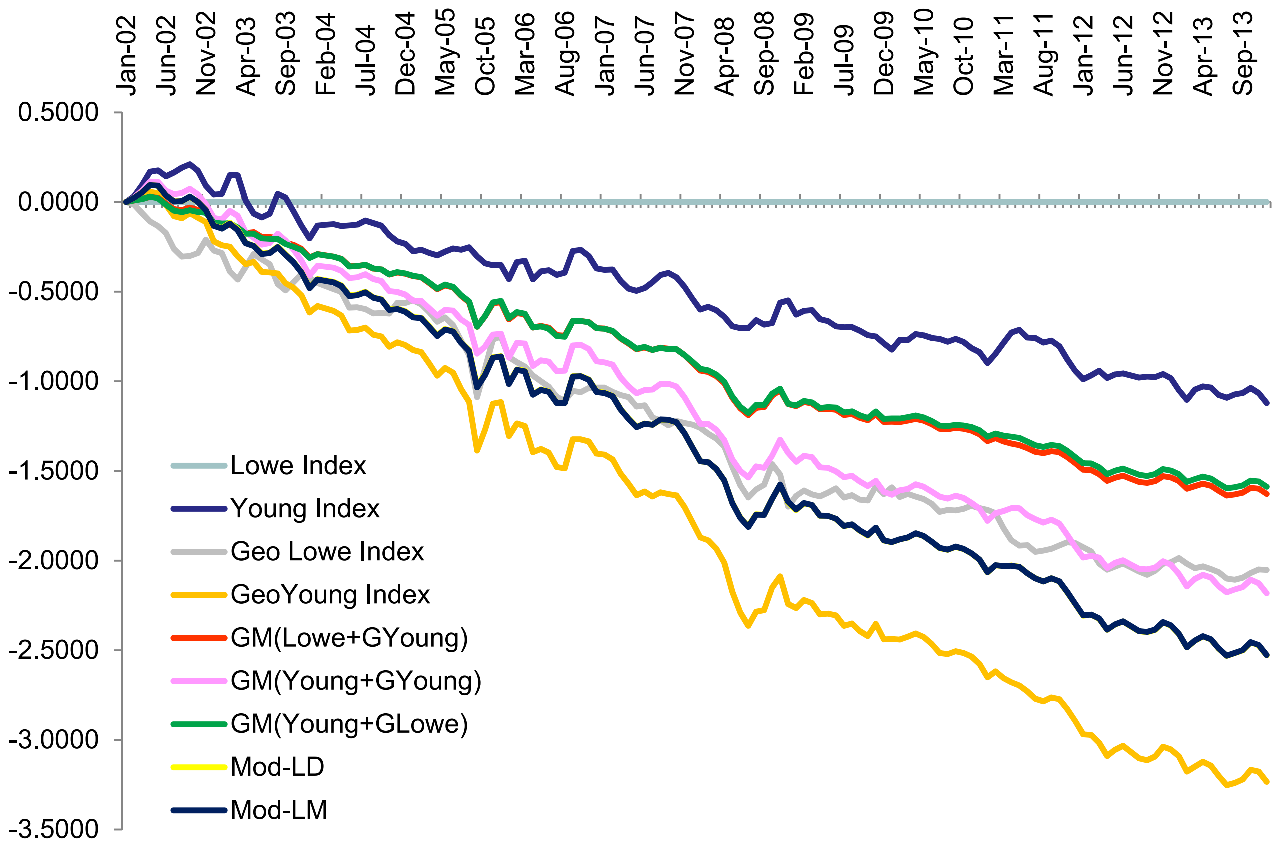
$$P^{t/0}_{GM\,(Lowe+GYoung)} = \left(P^{t/0}_{Lo}\right)^{1/2} \left(P^{t/0}_{GY}\right)^{1/2}$$

$$P^{t/0}_{GM\,(\mathrm{Young}+GLowe)} = \left(P^{t/0}_{Y}\right)^{1/2} \left(P^{t/0}_{GLo}\right)^{1/2}$$

$$P^{t/0}_{GM\,(\mathrm{Young}+GY)} = \left(P^{t/0}_{Y}\right)^{1/2} \left(P^{t/0}_{GY}\right)^{1/2}$$

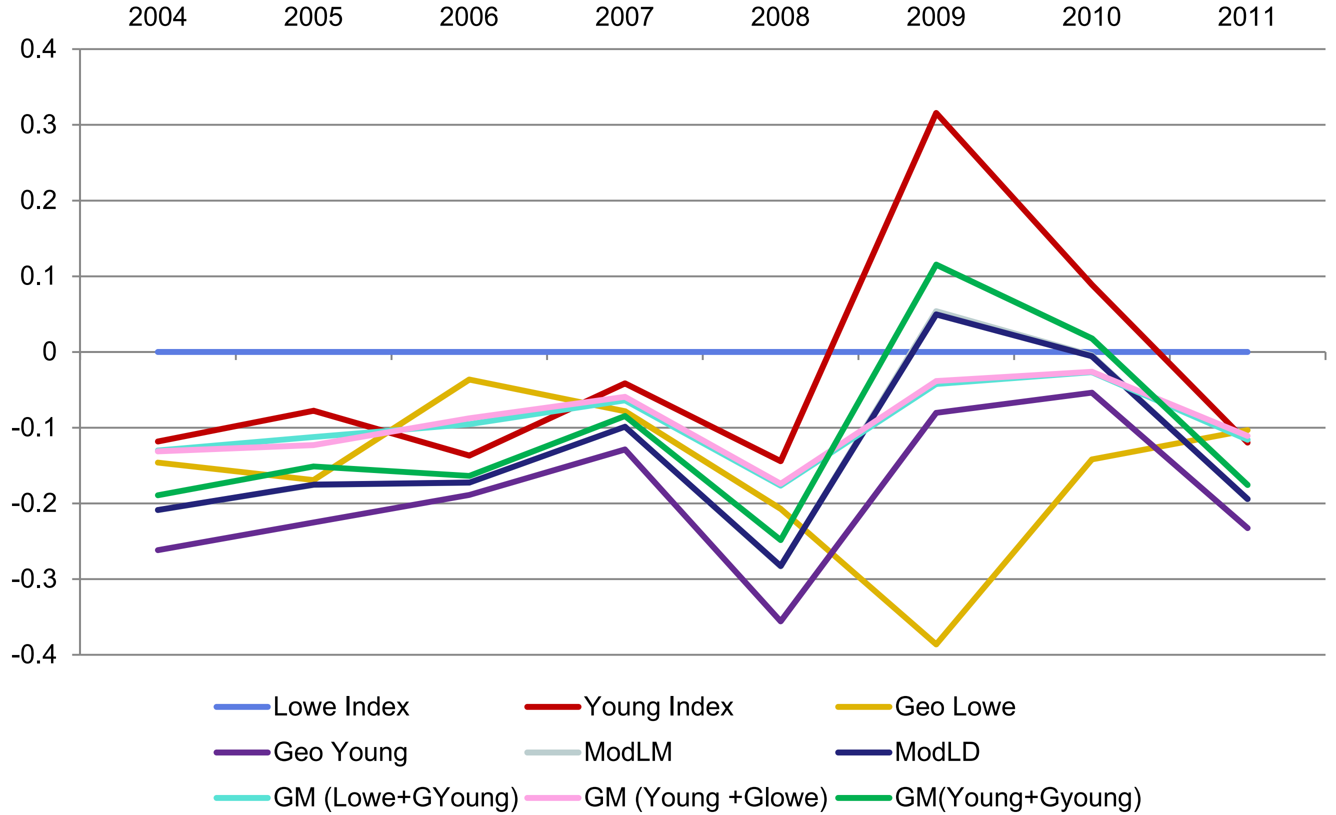
# Other Weighted Price Indices

- Differences in the index values between the Lowe index and the alternative price indices:



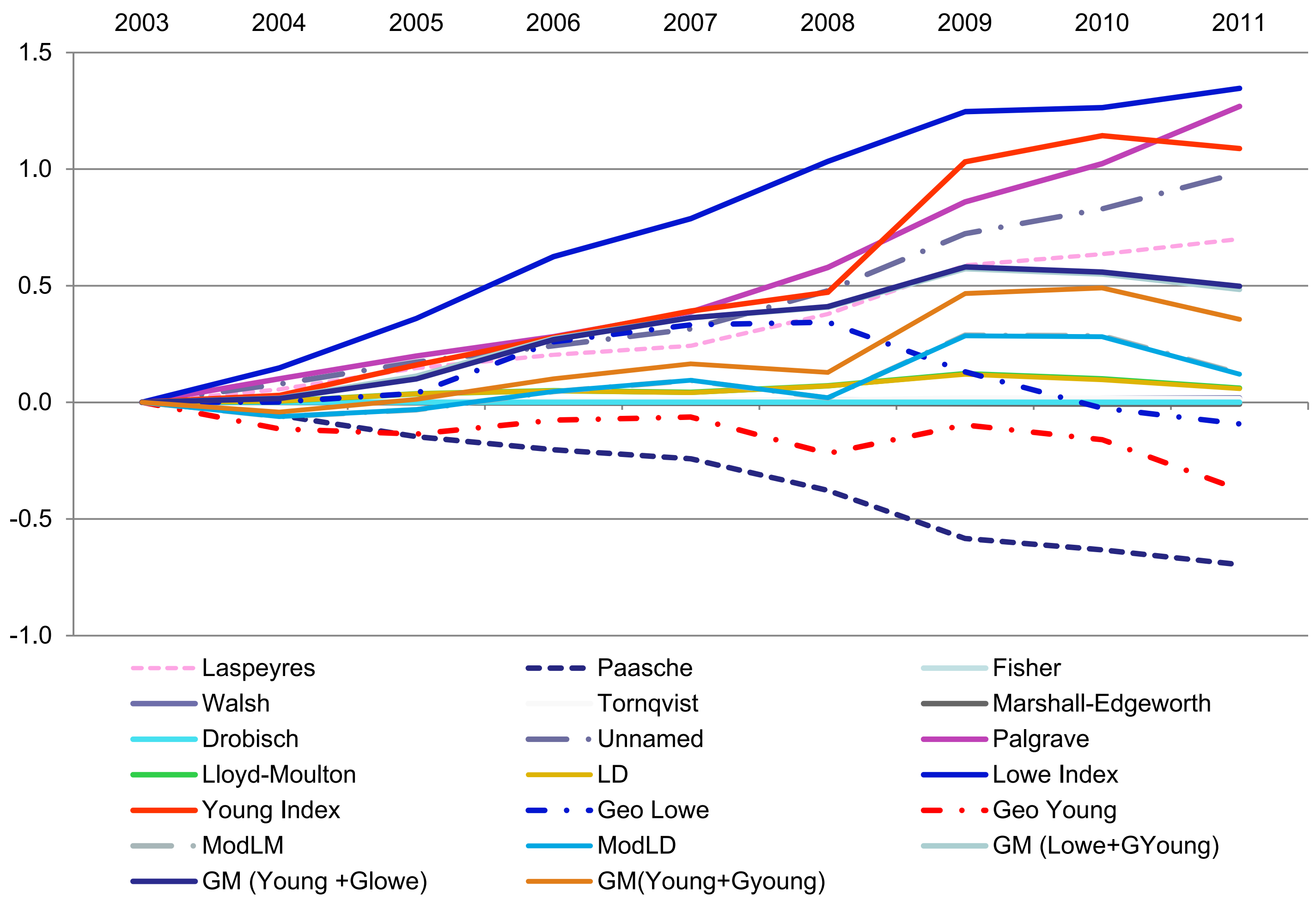
# Other Weighted Price Indices

- Differences in the annual inflation rates between the Lowe index and the alternative price indices:



# Comparison with the Target Index

- Differences in index values between the Fisher index and the other indices



# Comparison with the Target Index

| | Index (2003=100) | Index Difference | Average Annual Growth Rate | Growth Rate Difference |
|---|---|---|---|---|
| | 2003-2011 | | (%) | (%) |
| Fisher | 114.389 | 0.000 | 1.695 | 0.000 |
| Walsh | 114.408 | 0.019 | 1.697 | 0.002 |
| Törnqvist | 114.405 | 0.016 | 1.696 | 0.002 |
| M-E | 114.381 | -0.008 | 1.694 | -0.001 |
| Drobisch | 114.390 | 0.000 | 1.695 | 0.000 |
| Un-named | 115.374 | 0.984 | 1.804 | 0.109 |
| Laspeyres | 115.090 | 0.700 | 1.772 | 0.078 |
| Lloyd-Moulton | 114.451 | 0.061 | 1.701 | 0.007 |
| LD | 114.448 | 0.059 | 1.701 | 0.007 |
| Palgrave | 115.658 | 1.269 | 1.835 | 0.140 |
| Paasche | 113.693 | -0.696 | 1.617 | -0.078 |

# Comparison with the Target Index

| | Index (2003=100) | Index Difference | Average Annual Growth Rate | Growth Rate Difference |
|---|---|---|---|---|
| | 2003-2011 | | (%) | (%) |
| Fisher | 114.389 | 0.000 | 1.695 | 0.000 |
| Lowe | 115.736 | 1.346 | 1.844 | 0.149 |
| Young | 115.432 | 1.043 | 1.810 | 0.115 |
| Geometric Lowe | 114.297 | -0.092 | 1.684 | -0.010 |
| Geometric Young | 114.015 | -0.374 | 1.653 | -0.042 |
| Modified L-M index | 114.514 | 0.125 | 1.709 | 0.014 |
| Modified L-D index | 114.510 | 0.121 | 1.708 | 0.013 |
| GM (Lowe+ GY) | 114.873 | 0.484 | 1.748 | 0.054 |
| GM (Young+GLo) | 114.865 | 0.475 | 1.747 | 0.053 |
| GM (Young+GY) | 114.723 | 0.333 | 1.732 | 0.037 |

# Conclusions

- Superlative indices closely approximate each other;
- Most symmetrically weighted price indices are close approximates to each other, except for the Un-named price index;
- The comparison between the Paasche index and the Palgrave index implies that the use of current information can generate quite different results depending on how the data are used;
- The Lowe index yielded the highest upper-level substitution bias among the indices examined in this study;

# Conclusions (cont.)

- Among the price indices discussed in this study, the Modified LD and Modified Lloyd-Moulton indices tracked the selected target index most closely. They could be used to estimate the monthly substitution bias;
- Geometric Young index yielded a downward bias;
- The three L-D approximates reduced the substitution bias in the Lowe index;
- The geometric mean of the Lowe and Geometric Young index closely approximates the geometric mean of the Young and Geometric Lowe index.