A modification of the GEKS index when product turnover is high

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Abstract: Recent research on price measurement from scanner data has included comparisons of new methods such as the ITRYGEKS index (de Haan and Krsinich, 2014a) and the FEWS index (Krsinich, 2014) to the GEKS price index (Ivancic, Diewert and Fox, 2011). The consumer electronics scanner data set from which these findings were derived is characterized by a high turnover of products, which means that the bilateral price indices underlying the compilation of the GEKS price index are based on different sets of matched products. In this paper, a variant of the usual GEKS approach is proposed which aims to correct this imbalance. It differs from the usual GEKS by being defined as a chained index from t-1 to 1, with unmatched products between t-1 and t being omitted from the Törnqvist indexes. Empirical results show that this alternative approach tends to be less volatile, and to more closely match the multilateral time-dummy hedonic index than the GEKS, for the eight consumer electronics product categories we examine.

1. Introduction

Price measurement from scanner data requires new methods – high levels of product turnover mean that direct superlative indexes become quickly unrepresentative, and

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volatile prices and quantities cause high-frequency superlative indexes to suffer from chain-drift.

The Rolling Year (RY) GEKS of Ivancic *et al* (2011) reflects all matched products in the data while being unaffected by chain-drift. Recent findings by Krsinich (2014) on an apparent window-length biasing[3] in the GEKS led us to investigate a modification of the GEKS formula to deal with its asymmetry in the case of product turnover.

Consumer electronics have a high level of product turnover, due to rapid technological change. We have used New Zealand scanner data for eight consumer electronics product categories, from market research company GfK, to develop and test a modification of the GEKS index, called the 'intersection GEKS' (intGEKS) index which, for most of the eight product categories examined, is less volatile than the GEKS and sits closer to the multilateral time-dummy hedonic (TD) index which we use as a benchmark.

Product turnover is an essential feature of the methodological discussion in this paper, which becomes trivial if prices for the same products are available over the whole time window. In other words, the intGEKS equals the GEKS when there is no turnover of products.

2. Background

Krsinich (2014) showed that, when characteristics are not available[4] in the data to support explicit hedonic quality-adjustment, fixed-effects models are equivalent to fully-interacted time-dummy hedonic indexes where all characteristics defining the products are stated as categorical variables and included in the hedonic model. This is achieved by taking advantage of the longitudinal structure of the data.

When combined with a modified approach to splicing, which incorporates the index across the full estimation window, rather than just the most recent movement, this fixed-effects window-splice (FEWS) index is a non-revisable quality-adjusted price index, despite not explicitly incorporating any characteristics of the products.

[3] In fact, the apparent flattening with increasing window length was largely driven by an error in the weighting of the GEKS in Krsinich (2014).
[4] And even if characteristics are available.

The FEWS index was compared to the Imputation Törnqvist RYGEKS (ITRYGEKS) index of de Haan and Krsinich (2014a) which explicitly incorporates characteristics, and the RYGEKS of Ivancic et al (2011). It was found that the RYGEKS showed less rapid price decreases[5] than the FEWS and ITRYGEKS indexes for all product categories except DVD players and recorders, and microwaves. The RYGEKS also tended to be more volatile than either of the FEWS or ITRYGEKS indexes.

It was thought that the difference between the RYGEKS and the FEWS reflects two factors:

Firstly, unlike the FEWS index (and the ITRYGEKS index), the RYGEKS does not include the implicit price movements of new products entering into, and old products disappearing from, the market.

Secondly, it appeared that the RYGEKS index was very sensitive to the length of the estimation window, with increasing window lengths corresponding to systematically flatter GEKS indexes. This was the initial motivation for this research, but subsequent analysis showed that the flattening was in fact mainly due to an error in the weighting of the GEKS results in Krsinich (2014).

The process of investigating this apparent bias led to a focus on the asymmetry of the GEKS index in the case of product turnover. The modification of the GEKS developed to deal with this asymmetry performs better than the standard GEKS when compared to a multilateral time-dummy hedonic (TD) benchmark index.

The asymmetry of the GEKS index is dealt with by dropping from the calculation products which are unmatched between t-1 and t. The resulting 'intersection GEKS' index is explained in detail in the next section.

3. The intersection GEKS

Ivancic *et al* (2011) applied the GEKS method to scanner data, using a rolling window to result in a non-revisable index for production, the rolling year GEKS (RYGEKS).

[5] Note that in general results can go either way depending on data characteristics. For instance, Greenlees and McClelland (2010) found that on US scanner data for apparels, the RYGEKS was overly downward oriented. We hypothesize that this was a consequence of insufficiently overlapping samples.

The GEKS is a multilateral index whose main advantage is that it satisfies the circularity (or transitivity) requirement, thus making the results free of chain drift. It is derived by making bilateral comparisons between any two periods belonging to a fixed time window. The underlying bilateral price index should satisfy the time reversal test. In this paper, we use the Törnqvist price index, which can be seen as the square root of a Geometric Laspeyres and a Geometric Paasche price index. We denote by S_t the set of products[6] available in a given period t.

$$P_{t1,t2} = \sqrt[2]{P_{t1,t2}^{GL} * P_{t1,t2}^{GP}}, \tag{1}$$

where

$$P_{t1,t2}^{GL} = \prod_{i \in S_{t1} \cap S_{t2}} \left(\frac{p_i^{t2}}{p_i^{t1}}\right)^{s_i^{t1}} , \tag{2}$$

$$P_{t1,t2}^{GP} = \prod_{i \in S_{t1} \cap S_{t2}} \left(\frac{p_i^{t2}}{p_i^{t1}}\right)^{s_i^{t2}} . \tag{3}$$

Note that the compilations are solely based on price and quantity data from products which are available in both periods t1 and t2. The matched principle implies that new or disappearing products that are available in only one of the two comparison periods are ignored.

The GEKS can be calculated between any two time points within a given time window which ranges from period 1 to period K. For the purposes of the comparison to the intersection GEKS, we state it in terms of the index between t-1 and t:

$$P_{t-1,t[1;K]}^{GEKS} = \prod_{k=1..K} \left(P_{t-1,k} * P_{k,t}\right)^{\frac{1}{K}} \tag{4}$$

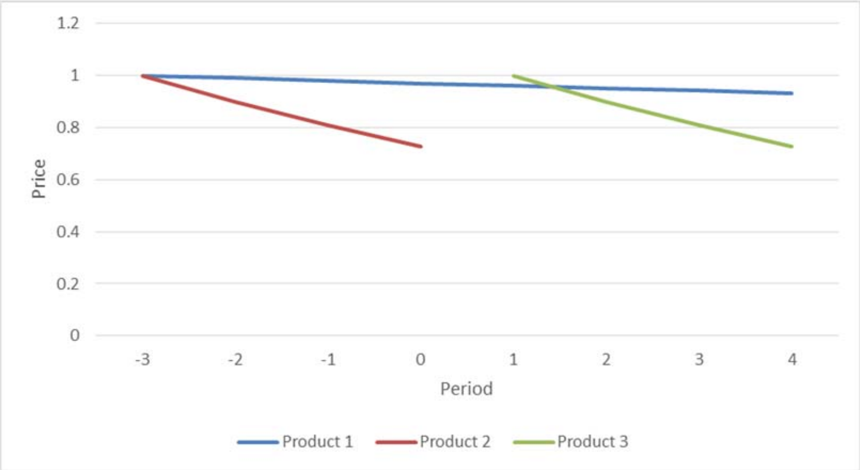
A limitation of the standard GEKS method when used for goods with high product turnover, is its asymmetry. For example, for the GEKS between periods t-1 and t, any unmatched products with respect to the periods t-1 and t but available in period k will

[6] We use the term ‘product’ rather than ‘item’ for consistency with recent papers by Krsinich.

be included in either the first bilateral comparison or the second bilateral comparison but not both.

To illustrate, consider the example with 3 products shown in figure 1.The first product is available in all periods and shows a regular price decline of 1% in each period. The two other products have shorter life cycles and sharper price decreases of 10% in each period. Whereas the second product is only available during the first four periods, the third product is only available during the last four periods. . For simplicity, we assume that in each period the two products that are available have a 50% share.

Figure 1. The example data



The GEKS price index compiled over this 8 period window consistently decreases during the first 4 periods, before increasing by 6.3% in period 1 compared to period 0 (see figure 3). The GEKS price index between period 1 and period 0 is defined as the geometric mean over the entire time window of the bilateral price indexes $P_{0,k}$ times $P_{k,1}$. In this example, the underlying bilateral indexes are defined on different sets of matched products. In the first half of the time window *(k=-3, -2, -1, 0)*, the price index $P_{0,k}$ is based on both products 1 and 2 while the price index $P_{k,1}$ only relies on product 1. Consequently, the first price index incorporates the stronger price decreases of product 2 which are not compensated for by the second price index. There is a similar imbalance in the second half of the time window *(k=1, 2, 3, 4)*: On the one hand $P_{0,k}$ is only derived from product 1, but on the other hand $P_{k,1}$ takes into account products 1 and 3.

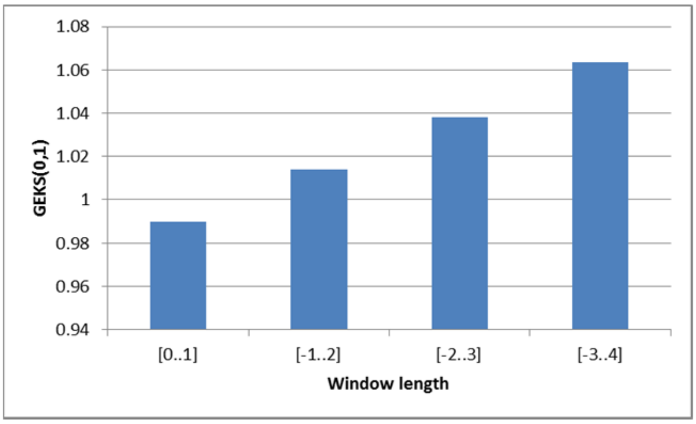
In the end (see table 1), the asymmetry caused by the disappearance and appearance of products 2 and 3 adjusts the result of the GEKS upwards. This result can be surprising from a short-term perspective because product 1, the only product available in both periods 0 and 1, is decreasing by 1%.

Table 1. The GEKS index between periods 0 and 1

k	$P_{0,k}$	$P_{k,1}$	$P_{0,k}$ * $P_{k,1}$
Period -3	1.189	0.961	1.142
Period -2	1.122	0.970	1.089
Period -1	1.059	0.980	1.038
Period 0	1.000	0.990	0.990
Period 1	0.990	1.000	0.990
Period 2	0.980	1.059	1.038
Period 3	0.970	1.122	1.089
Period 4	0.961	1.189	1.142
		$P^{GEKS}_{0,1,[-3;4]}$	1.063

The last column of table 1 also shows that the product term $P_{0,k}$ * $P_{k,1}$ is deviating more from the bilateral price index $P_{0,1}$ when period k is further away from the comparison periods 0 and 1. In other words, enlarging the window length will worsen the imbalance created by products 2 and 3 being partly unavailable (see figure 2).

Figure 2. The GEKS index between periods 0 and 1 as a function of the window length



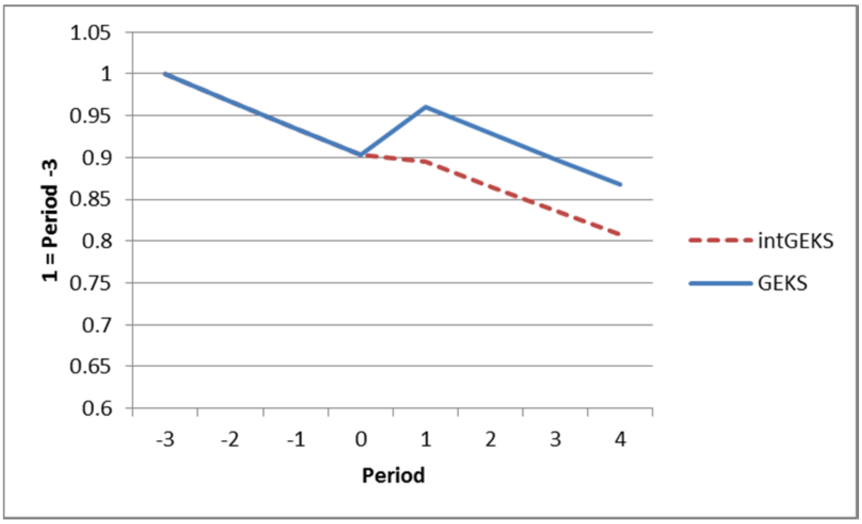
A variation of the GEKS approach which deals with this asymmetry is what we're calling the 'intersection GEKS', or intGEKS. This index is defined as a chained index. For the intGEKS between t-1 and t, the bilateral Törnqvists that are incorporated into the intGEKS are based on products matching between t-1, t and k. In other words, the products which are new or disappearing between t-1 and t are left out of the calculation entirely, rather than being asymmetrically included, as in the GEKS.

So, using the example data, only product 1 is available in the two comparison periods 0 and 1. In such a trivial case, the bilateral price indexes are fully transitive ($P_{0,k} * P_{k,1} = P_{0,1}$). Consequently, the intersection GEKS between periods 4 and 5 will simply be equal to the product 1 price change.

Table 2. The intGEKS index between periods 0 and 1

k	$P_{0,k}$	$P_{k,1}$	$P_{0,k} * P_{k,1}$
Period -3	1.031	0.961	0.990
Period -2	1.020	0.970	0.990
Period -1	1.010	0.980	0.990
Period 0	1.000	0.990	0.990
Period 1	0.990	1.000	0.990
Period 2	0.980	1.010	0.990
Period 3	0.970	1.020	0.990
Period 4	0.961	1.031	0.990
		$P_{0,1,[-3;4]}^{intGEKS}$	0.990

Figure 3. The GEKS and the intGEKS price index on the example data



Formally, the compilation of the price change between two adjacent time periods t-1 and t will be restricted only to the set of matched products over the three periods t-1, t and k. We call this the "intersection set":

$$M_{t-1,t}^{k} = S_{t-1} \cap S_k \cap S_t. \tag{5}$$

The intGEKS price index then averages these restricted bilateral comparisons over all the periods contained in the time window:

$$P_{t-1,t[1;K]}^{intGEKS} = \prod_{k=1..K} \left(P_{t-1,k}^{M_{t-1,t}^{k}} * P_{k,t}^{M_{t-1,t}^{k}} \right)^{\frac{1}{K}}. \tag{6}$$

The bilateral Törnqvist price index $P_{t-1,k}$ takes into account those products which are available in t-1 and in k, whereas in this adjusted framework, only products available in periods t-1, k and t are taken into account to compile $P_{t-1,k}^{M_{t-1,t}^{k}}$. Similarly, the price index $P_{k,t}$ is based on the set of matched products between k and t, but for the compilation of $P_{k,t}^{M_{t-1,t}^{k}}$ products must be available in all three periods k, t and t-1.

With the matched principle applying to both approaches, neither the GEKS nor the intGEKS price index take into account products which are sold in only one of the periods t-1, t or k.

If exactly the same products are available in t-1 and in t, then the matched products between t-1 and k are identical to the matched products between k and t. That is why identical results are obtained:

$$P_{t-1,t[1;K]}^{GEKS} = P_{t-1,t[1;K]}^{intGEKS} \qquad if\ S_t = S_{t-1}. \tag{7}$$

If the set of products is constant over the entire time window [1;K], then there is no difference between the GEKS and the intersection GEKS.

Note that the estimation window lengths possible for the GEKS and intGEKS indexes are constrained by the component bilateral Törnqvists that feed into them – that is, if there are any pairs of months within the estimation window for which there are no matched products, then the GEKS will be undefined. The possible estimation windows for the intGEKS are even more tightly constrained, as they require matched products within the triples of months t-1,t and k.

It is well known that the GEKS index is transitive. The transitivity property is one of the main reasons for applying the GEKS method to scanner data, because it prevents chain drift.

The intGEKS, however, violates transitivity. To see why, consider the following situation:

$$P_{1,2[1;K]}^{intGEKS} * P_{2,3[1;K]}^{intGEKS} = \prod_{k=1..K}\left(P_{1,k}^{M_{1,2}^{k}} * P_{k,2}^{M_{1,2}^{k}} * P_{2,k}^{M_{2,3}^{k}} * P_{k,3}^{M_{2,3}^{k}}\right)^{\frac{1}{K}}. \qquad (8)$$

Now consider the direct comparison between period 3 and the base period 1:

$$P_{1,3[1;K]}^{intGEKS} = \prod_{k=1..K}\left(P_{1,k}^{M_{1,3}^{k}} * P_{k,3}^{M_{1,3}^{k}}\right)^{\frac{1}{K}}. \qquad (9)$$

For the transitivity requirement to be satisfied, we need to have that:

$$P_{1,k}^{M_{1,2}^{k}} = P_{1,k}^{M_{1,3}^{k}} \quad , \qquad (10)$$

$$P_{k,3}^{M_{2,3}^{k}} = P_{k,3}^{M_{1,3}^{k}} \quad , \qquad (11)$$

$$P_{k,2}^{M_{1,2}^{k}} = \frac{1}{P_{2,k}^{M_{2,3}^{k}}} \quad . \qquad (12)$$

However, because the compilation of the bilateral price indexes may be based on different sets of products, we have no guarantee that these equalities hold.

Transitivity breaks down if the intersection sets $M_{1,2}^{k}$, $M_{1,3}^{k}$ and $M_{2,3}^{k}$ are very different. However, in practice, the set of products available in adjacent periods may only be gradually evolving and consequently the degree of violation of the transitivity property can be expected, in the short-term, to be small. This explanation may be less valid when applied to periods which are further apart. However, in a context of high product turnover rates, the relevance of the transitivity property can also be questioned.

Moreover, the rolling versions of the GEKS which are typically used in practical applications also violate the transitivity property. Despite this formal flaw, it is nevertheless recognized that rolling GEKS price indexes provide results that are almost free of chain-drift.

The multilateral time-dummy hedonic price index is transitive and therefore, by definition, free of chain-drift. Empirical results for consumer electronics, shown in section 6, suggest that any 'chain-drift' due to the non-transitivity of the intGEKS is not significant.

Note that the intGEKS exploits less data than the usual GEKS. Only the products belonging to the intersection set $M_{t-1,t}^{k}$ enter the compilation. To better understand the difference between the GEKS and the intGEKS, we need to look at disappearing and new products. The set of disappearing products consists of those products which are available in t-1 and not available in t.

$$D_{t-1,t}^{k} = \{i | i \in S_{t-1} \; and \; i \in S_k \; and \; i \notin S_t \}. \qquad (13)$$

The set of new products consists of those products which are not available in t-1 and are available in t.

$$N_{t-1,t}^{k} = \{i | i \notin S_{t-1} \; and \; i \in S_k \; and \; i \in S_t \}. \qquad (14)$$

As expected, the difference between the two approaches is driven by the price dynamics of the disappearing and new products which are included in the GEKS compilation via the bridge period k, but are excluded from the intGEKS compilation.

Formally, it is shown in appendix 1 that:

$$\frac{P^{intGEKS}_{t-1,t[1;K]}}{P^{GEKS}_{t-1,t[1;K]}} = \prod_{k=1..K} \left(\left(\frac{P^{GL|M^k_{t-1,t}}_{t-1,k}}{P^{GL|D^k_{t-1,t}}_{t-1,k}} \right)^{0.5 s^{t-1|M_{t-1,k}}_{D^k_{t-1,t}}} \cdot \left(\frac{P^{GP|M^k_{t-1,t}}_{t-1,k}}{P^{GP|D^k_{t-1,t}}_{t-1,k}} \right)^{0.5 s^{k|M_{t-1,k}}_{D^k_{t-1,t}}} \right.$$
$$\left. \cdot \left(\frac{P^{GL|M^k_{t-1,t}}_{k,t}}{P^{GL|N^k_{t-1,t}}_{k,t}} \right)^{0.5 s^{k|M_{k,t}}_{N^k_{t-1,t}}} \cdot \left(\frac{P^{GP|M^k_{t-1,t}}_{k,t}}{P^{GP|N^k_{t-1,t}}_{k,t}} \right)^{0.5 s^{t|M_{k,t}}_{N^k_{t-1,t}}} \right)^{\frac{1}{K}} . \qquad (15)$$

The first and third terms of expression 15 are ratios of geometric Laspeyres price indexes and the second and fourth terms are ratios of geometric Paasche price indexes. In the first two terms, period k is compared to period t-1. On the numerator, the comparison is restricted to products belonging to $M^k_{t-1,t}$ whereas on the denominator the comparison is restricted to products belonging to $D^k_{t-1,t}$. Similarly, in the last two terms of the expression, period t is compared to period k. On the numerator, the price comparison is restricted to products belonging to $M^k_{t-1,t}$ whereas on the denominator the comparison is restricted to products belonging to $N^k_{t-1,t}$.

The weight $s^{t-1|M_{t-1,k}}_{D^k_{t-1,t}}$ (or $s^{k|M_{t-1,k}}_{D^k_{t-1,t}}$) corresponds to the share in period t-1 (or in period k) of the products belonging to $D^k_{t-1,t}$, relative to the matched products between t-1 and k.

Similarly, the weight $s^{k|M_{k,t}}_{N^k_{t-1,t}}$ (or $s^{t|M_{k,t}}_{N^k_{t-1,t}}$) corresponds to the share in period k (or in period t) of products belonging to $N^k_{t-1,t}$, relative to the matched products between k and t.

This analytical expression shows that the GEKS and the intGEKS lead to identical results if the following two conditions hold. First, when comparing period t-1 and period k, the price dynamics of the matched sample and of the disappearing sample are the same. Second, when comparing period k and period t, the price dynamics of the matched sample and of the new sample are the same. Identical results may also be obtained if the deviation caused by the disappearing products is cancelled out by the deviation caused by the new products.

However in practice, the two indexes are likely to differ. Consider for instance the example shown at the beginning of this section where the disappearing and new products correspond to products with short life-cycles and important price decreases. The price change in period k compared to period t-1 (k<t-1) is then likely to be smaller for the matched products than for the disappearing products:

$$\frac{P_{t-1,k}^{GL/M_{t-1,t}^{k}}}{P_{t-1,k}^{GL/D_{t-1,t}^{k}}} < 1, \tag{16}$$

$$\frac{P_{t-1,k}^{GP/M_{t-1,t}^{k}}}{P_{t-1,k}^{GP/D_{t-1,t}^{k}}} < 1. \tag{17}$$

Similarly, the price change in period t compared to period k (k>t) is likely to be smaller for the matched products than for the new products:

$$\frac{P_{k,t}^{GL|M_{t-1,t}^{k}}}{P_{k,t}^{GL|N_{t-1,t}^{k}}} < 1, \tag{18}$$

$$\frac{P_{k,t}^{GP|M_{t-1,t}^{k}}}{P_{k,t}^{GP|N_{t-1,t}^{k}}} < 1. \tag{19}$$

If the inequalities 16-19 hold, it then follows from equation 15 that the aggregate price change of t compared to t-1 will be smaller for the intGEKS than for the GEKS, ie $P_{t-1,t[1;K]}^{intGEKS} < P_{t-1,t[1;K]}^{GEKS}$.

4. The Multilateral Time-Dummy Hedonic index

We consider a semi-log hedonic regression which links prices to their characteristics and which includes dummy variables representing the different time periods. In this framework, the data is pooled over the entire time window.

$$\ln(p_i^t) = \alpha + \sum_{t=2,..,K} \delta_t D_{it} + \sum_z \beta_z X_{iz} + \varepsilon_{it} \quad \forall\, i\ \in S_t, \forall\, t = 1, 2, \dots, K \tag{20}$$

The variable D_{it} equals 1 if the product *i* belongs to period *t* and D_{it} equals 0 otherwise. We use as regression weights the shares s_i^t of each product within each period. The resulting price index is then derived from the estimates of the time dummy variables.

$$P_{1,t[1;K]}^{TD} = \exp(\delta_t). \tag{21}$$

We use the multilateral time-dummy hedonic (TD) index as our benchmark index because it explicitly includes all characteristics whereas the GEKS and the intGEKS are derived only from prices without any explicit quality-adjustments. Unlike price indices based on the matched principle, the TD also reflects price movements of new and disappearing products. Moreover the TD is a transitive price index which consequently does not suffer from chain drift.

De Haan and Krsinich (2014b) showed that the TD is a geometric version of a 'quality-adjusted unit value index'.

$$P_{1,t[1;K]}^{TD} = \frac{\prod_{i\in S^t} \left(\widetilde{p_i^t}\right)^{s_i^t}}{\prod_{i\in S^1} \left(\widetilde{p_i^1}\right)^{s_i^1}} \quad , \tag{22}$$

where $\widetilde{p_i^t}$ and $\widetilde{p_i^1}$ are quality adjusted prices with respect to a reference product.

We formulate the intGEKS in terms of the TD to determine what drives the difference between both approaches:

$$\frac{P_{t-1,t[1;K]}^{intGEKS}}{P_{t-1,t[1;K]}^{TD}} = \prod_{k=1..K} \left(\frac{\left(\dfrac{\prod_{i\in M_{t-1,t}^k} (p_i^t)^{0.5\left(s_i^{t|M_{t-1,t}^k} + s_i^{k|M_{t-1,t}^k}\right)}}{\prod_{i\in S^t} \left(\widetilde{p_i^t}\right)^{s_i^t}} \right)}{\left(\dfrac{\prod_{i\in M_{t-1,t}^k} (p_i^{t-1})^{0.5\left(s_i^{t-1|M_{t-1,t}^k} + s_i^{k|M_{t-1,t}^k}\right)}}{\prod_{i\in S^{t-1}} \left(\widetilde{p_i^{t-1}}\right)^{s_i^{t-1}}} \right)} \cdot \prod_{i\in M_{t-1,t}^k} \left(p_i^k\right)^{0.5\left(s_i^{t-1|M_{t-1,t}^k} - s_i^{t|M_{t-1,t}^k}\right)} \right)^{\frac{1}{K}} \tag{23}$$

We can recognize in this expression that the quality-adjusted average price over all products available in a given period is compared to a weighted average price in that same period over the products belonging to the intersection set. The weights which are used to average the prices of the intersection set take into account the product shares in that given period and in period k. The last term combines prices in period k with the difference of the product shares between period t-1 and t.

It follows that if the average price in the intersection set compared to the average price of the whole sample in period t is identical to the same comparison made in period t-1 and if the product shares in t - 1 and in t with respect to the intersection set are the same, then the intGEKS and the TD provide identical results. The first condition suggests that the products in the intersection set are sufficient to estimate the quality-adjusted average price of all the products available in a given period up to a fixed factor. The second condition is likely to add a small but hopefully negligible noise to the comparison with the TD.

In the empirical results presented in section 6, we find that the intGEKS matches the benchmark time-dummy hedonic index relatively closely. The close correspondence of the intGEKS and the TD implies that the bias due to not reflecting the price movements of new and disappearing products in the price index is less significant than previously thought (for example by Ivancic *et al* 2011 and de Haan and Krsinich 2014a).

5. Data

Statistics New Zealand has been using scanner data for consumer electronics products from market research company GfK for a number of years, to inform expenditure weighting in the CPI. This data is close to full-coverage of the New Zealand market.

For research purposes, Statistics New Zealand purchased a more detailed dataset for the three years from mid 2008 to mid 2011 for eight product categories: camcorders; desktop computers; digital cameras; DVD players and recorders; laptop computers; microwaves; televisions; and portable media players. Monthly sales values and quantities are disaggregated by brand, model and around 40 characteristics. Microwaves are not really a ‘consumer electronics’ product but, as a product with less rapid technological change, they provide a useful comparison for the performance of different price index methods.

Consumer expenditures on the eight product categories differ substantially. Looking at the average of their monthly expenditure shares across the three years from mid-2008 to mid-2011, televisions are by far the most important product (44%), followed by laptop computers (26%). The average expenditure share of desktop computers (6%) is only one fifth of that of laptop computers. The other products’ average shares range from 8% for digital cameras to 2% for camcorders.

For confidentiality reasons, any brand where a single retailer has a share of more than 95% of total sales (except for microwaves, which has a threshold of 99%) is renamed to ‘tradebrand’ in GfK’s output system; similarly at the model level when a single retailer accounts for more than 80% of the sales of that model.

We define a ‘product’ as a unique combination of brand, model and the full set of characteristics available in the data. This can be seen as equivalent to the ‘barcode’ because it corresponds to the full level of detail on characteristics of the products. Note, however, that the data is aggregated across outlets. The service associated with particular outlets can be viewed as part of the total quality of a product, so any change in the composition of the sample in terms of outlets should ideally be controlled for. We are not able to do this.

Table 3 shows the average monthly number of distinct products in the data for each category. Camcorders have the least, at 88, while laptop computers have the highest average monthly number of distinct products across the 36 months we have data for – at 432.

Table 3. Average monthly number of distinct products in each product category.

Category	Products
Camcorders	88
Desktop computers	150
Digital cameras	289
DVD players and recorders	202
Laptop computers	432
Microwaves	152
Portable media players	161
Televisions	341

This data is characterized by a high turnover of products. Table 4 shows that, on average, only 42% to 55% of products are matched between two adjacent periods. This means that around half of the products are available in only one of two adjacent periods.

Table 4. Average monthly rates of new, disappearing and matched products[7]

Product	new	disappearing	matched
camcorders	27%	27%	46%
desktops	29%	29%	42%
digcamera	25%	25%	49%
dvds	25%	25%	50%
laptops	29%	29%	43%
microwaves	22%	23%	55%
portmedia	24%	25%	52%
television	24%	23%	53%

[7] Note – these are not quantity or expenditure weighted

Appendix 2 shows an artificial example of the data, for digital cameras. Note that all characteristics are expressed as categorical. Even numeric-type characteristics will only take a discrete number of values in the data – any change in a characteristic corresponds to a new product.

6. Empirical results

6.1 Comparison of the methods on maximum-lengthed estimation windows

Figure 4 to 6 show the intersection GEKS compared to the standard GEKS, the multilateral time-dummy hedonic (TD) and the chained Törnqvist indexes. We show these four indexes for digital cameras, DVD players and recorders, and portable media players because for these three product categories the impact of the ‘asymmetry bias’ in the GEKS is the most pronounced.

The GEKS and intGEKS indexes are calculated for the full 36-month window for digital cameras and DVD players and recorders, and for a shorter 30-month window for portable media players. This is because, if there are bilateral periods within the period which have no matching products, the bilateral Törnqvist indexes cannot be estimated, and therefore the GEKS and intGEKS are undefined.[8]

[8] Note also that, for this reason, the intGEKS is undefined for the last month of the 30 month window for portable media players. Rather than re-estimating TD, GEKS, and intGEKS indexes on a 29-month window, we have retained the 30-month window.

Figure 4. Comparison of methods for digital cameras

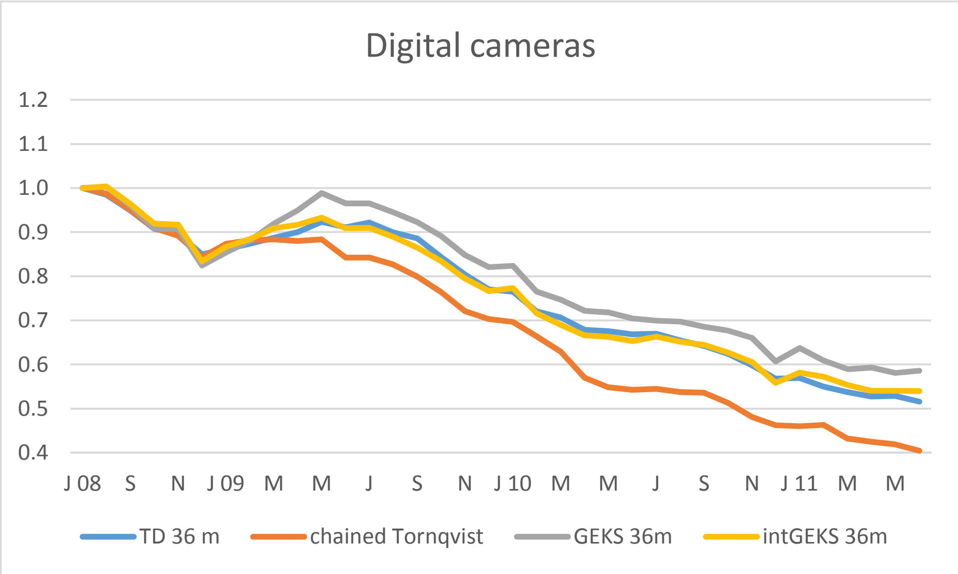


Figure 5. Comparison of methods for DVD players and recorders

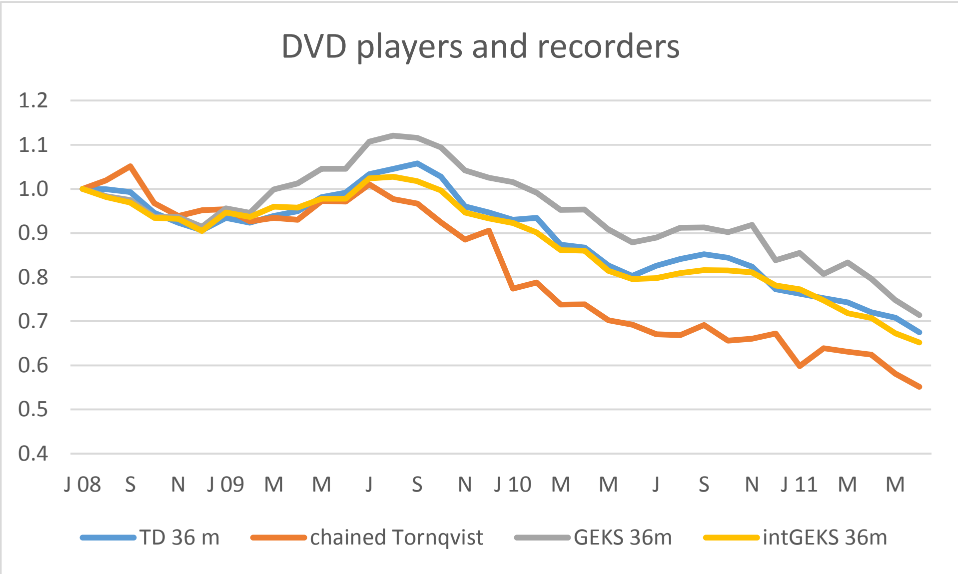
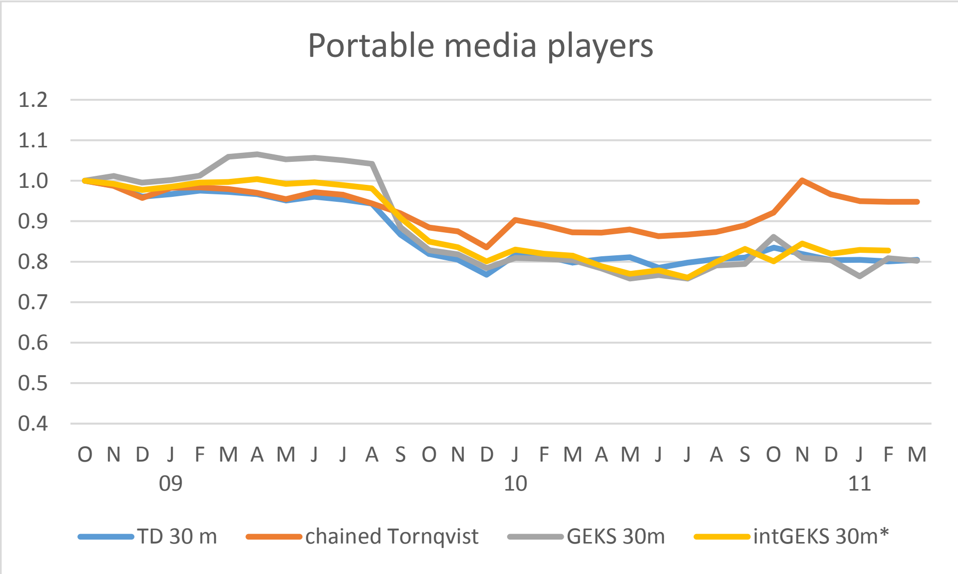


Figure 6. Comparison of methods for portable media players



The intersection GEKS matches the time-dummy (TD) hedonic index more closely than the GEKS index does, across the 3 year period, for all three of these product categories. For digital cameras and DVD players and recorders the GEKS index sits above both the intGEKS and the TD for most of the 3 year period, while for portable media players the GEKS sits higher in the first year, before returning to similar levels for the next two years.

The chained Törnqvist has a downwards chain-drift bias for both digital cameras and DVD players and recorders, and the bias is upwards for portable media players.

The difference between the intGEKS and the TD reflects the fact that the TD incorporates the implicit price movements of new and disappearing products, while the intGEKS doesn't. Although this omission may be behind some short-term differences between the intGEKS and the TD, it does not appear to result in any systematic biasing over time for these product categories – at least over the 36- and 30-month windows examined.

Figures 7 to 9 below show the four indexes in terms of both monthly and annual percentage movements.

Figure 7. Monthly and annual movements for digital cameras

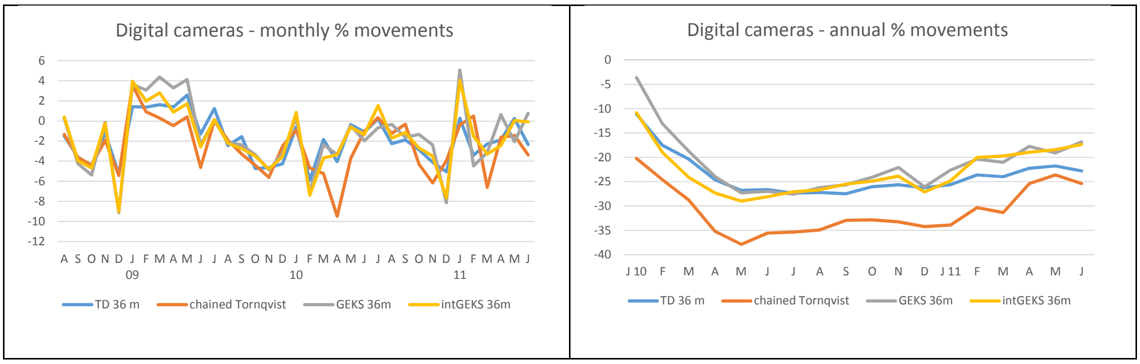


Figure 8. Monthly and annual movements for DVD players and recorders

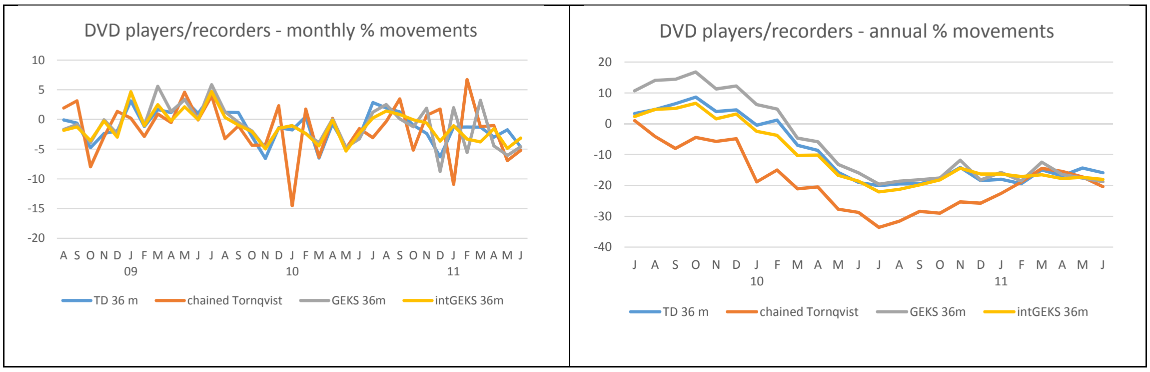
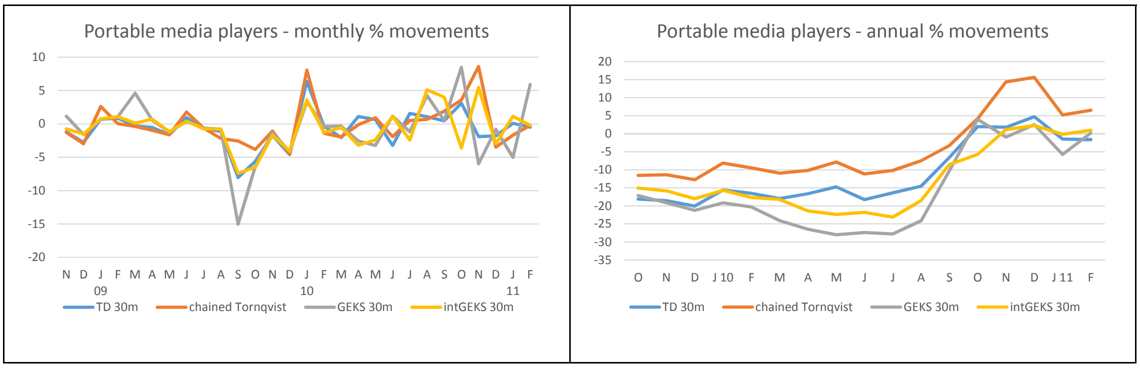


Figure 9. Monthly and annual movements for portable media players



The monthly percentage movements show that the GEKS and the chained Törnqvist tend to be more volatile than the intGEKS and TD indexes.

The annual percentage movements largely smooth out the seasonal variation and show the general direction of the biases in the chained Törnqvist and GEKS indexes. The chain-drift of the Törnqvist is more significant than the asymmetry-bias of the GEKS for all three product categories. For digital cameras and DVD players and recorders the chained-Törnqvist is biased downwards while the GEKS is biased upwards. For portable media players the situation is reversed, with the chained Törnqvist biased upwards and the GEKS biased downwards for much of the 3-yearly period.

Figure 10 summarises the volatility of the four indexes for all eight product categories in terms of average absolute monthly percentage movements.

Figure 10. Average absolute monthly percentage movements

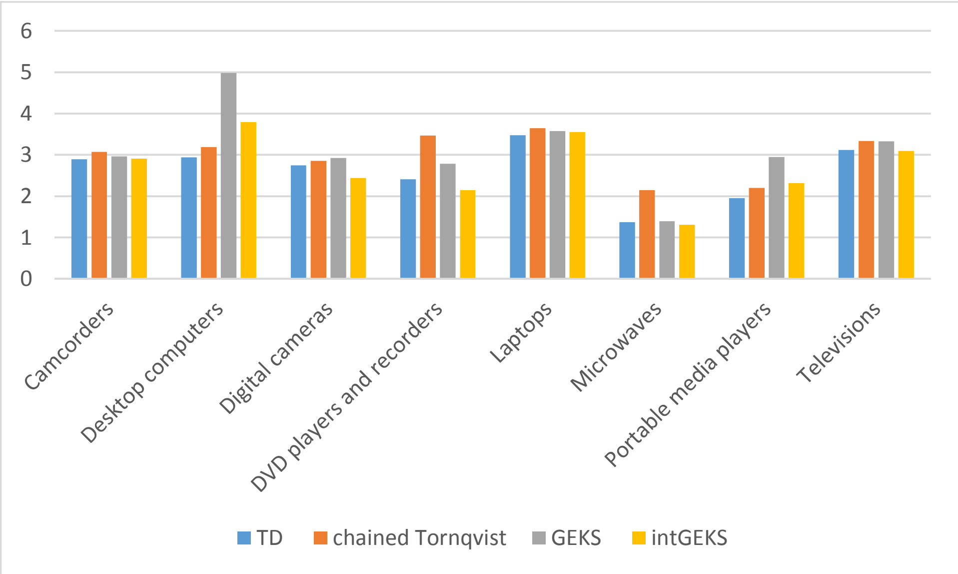
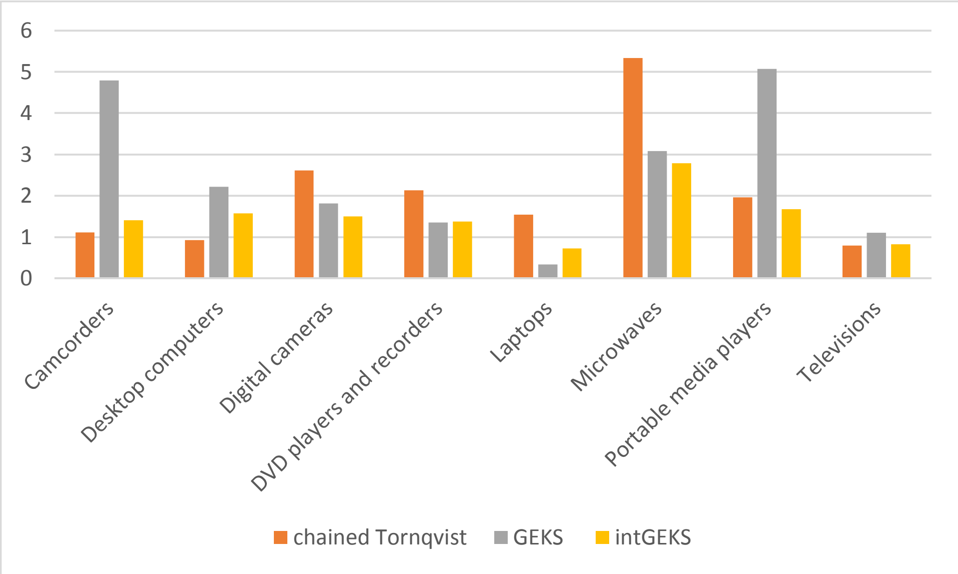


Figure 10 shows that for all the eight products, the average monthly movements of the GEKS index are greater than the intGEKS.

Figure 11 summarises the difference from the benchmark TD index of each of the chained Törnqvist, GEKS and intGEKS indexes in terms of the average relative difference of the monthly movements from that of the TD index.

Figure 11. Relative difference from TD monthly percentage movement



With the exception of DVD players and recorders, and laptops, the intGEKS is closer to the TD than the GEKS is, significantly so for both camcorders and portable media players.

6.2 Comparison of methods on 13-month windows

In figure 12, we show the intGEKS index for all eight product categories, compared to the standard GEKS, the multilateral time-dummy hedonic (TD) index and the monthly chained Törnqvist. These are estimated on a 13-month window in the middle of the 3-year study period, as this is the length of estimation window that is most likely to be used in production for non-revisable indexes such as the rolling year (RY) GEKS or the RYTD.

Figure 12. TD, chained Törnqvist, GEKS, and intGEKS – eight product categories

With these shorter estimation windows, the biases in the chained Törnqvist and GEKS indexes accumulate less and the four indexes are relatively similar, other than for desktop computers, laptops and portable media players.

To compare the indexes more precisely, table 5 gives the index numbers at the end of the 13-month estimation windows, with Table 6 showing the corresponding differences from the TD benchmark for each of the chained-Törnqvist, GEKS and intGEKS.

Table 5. Index numbers at July 2010 (based at July 2009)

	TD(13m)	chained Tornqvist	GEKS(13m)	intGEKS(13m)
Camcorders	0.608	0.635	0.600	0.614
Desktop computers	0.718	0.733	0.712	0.719
Digital cameras	0.682	0.676	0.673	0.682
DVD players and recorders	0.748	0.737	0.744	0.735
Laptops	0.672	0.739	0.650	0.676
Microwaves	0.917	0.903	0.922	0.917
Portable media players	0.845	0.877	0.746	0.812
Televisions	0.583	0.594	0.571	0.586

Table 6. Difference from TD benchmark

	chained Tornqvist	GEKS(13m)	intGEKS(13m)
Camcorders	0.027	-0.008	**0.006**
Desktop computers	0.015	-0.006	**0.001**
Digital cameras	-0.006	-0.009	**0.000**
DVD players and recorders	-0.010	**-0.004**	-0.013
Laptops	0.067	-0.022	**0.004**
Microwaves	-0.014	0.006	**0.000**
Portable media players	**0.032**	-0.099	-0.033
Televisions	0.011	-0.012	**0.003**

For six of the eight product categories, the intGEKS matches the TD most closely by the end of 13 months, while the GEKS and the chained Törnqvist match most closely for one product category each – the GEKS is closest to the TD index for DVD players and recorders, and the chained-Törnqvist is closest to the TD for portable media players.

7. Conclusion

The GEKS appears to be biased in the case of high product-turnover by theasymmetric inclusion of new and disappearing products' prices and quantities.

By excluding the unmatched products in a modification of the GEKS which we call the intersection GEKS, this bias due to product-turnover is removed, resulting in an index that sits very close to a benchmark multilateral time-dummy hedonic index – which explicitly incorporates the characteristics of products into the hedonic quality-adjustment, and is free of chain-drift.

After adjusting for this 'asymmetry-bias' of the GEKS, the remaining bias in the intGEKS due to not reflecting the implicit price movements of unmatched products is less significant than has been previously concluded in the literature, for example in de Haan and Krsinich (2014a).

The intGEKS is defined as a chained index and so, unlike the GEKS, the intGEKS is not transitive. Empirically though, we show that the chain-drift of the intGEKS is insignificant for the eight consumer electronics products analysed. Arguably, transitivity is a less relevant or desirable property in the presence of high product turnover such as we find for these consumer electronics products.

The intGEKS may therefore be a viable method for products where we have scanner data with no characteristics. Although it doesn't reflect the implicit price movements of new/disappearing products, this bias appears to be less significant than previously thought and, unlike the FEWS index, the intGEKS requires no regression modelling and may be easier to explain and/or justify to users.

Appendix 1 :

In the GEKS price index, the bilateral comparisons are based either on the matched sets between periods t-1 and k or the matched sets between periods k and t:

$$M_{t-1,k} = S_{t-1} \cap S_k \quad , \tag{A.1}$$

$$M_{k,t} = S_k \cap S_t . \tag{A.2}$$

In the intGEKS, the comparisons are based on the intersection set, which is a subset of the above defined matched sets:

$$M_{t-1,t}^{k} = S_{t-1} \cap S_k \cap S_t \tag{A.3}$$

The following relationships hold with respect to the set of disappearing products and the set of new products:

$$M_{t-1,k} = M_{t-1,t}^{k} \cup D_{t-1,t}^{k} \quad , \tag{A.4}$$

$$M_{k,t} = M_{t-1,t}^{k} \cup N_{t-1,t}^{k} . \tag{A.5}$$

Let us compare the Törnqvist price index when derived either from products belonging to $M_{t-1,k}$ or products belonging to $M_{t-1,t}^{k}$.

$$\frac{P_{t-1,k}^{M_{t-1,t}^{k}}}{P_{t-1,k}^{M_{t-1,k}}}$$

$$= \frac{\prod_{i\in M_{t-1,t}^{k}}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5s_i^{t-1|M_{t-1,t}^{k}}} \prod_{i\in M_{t-1,t}^{k}}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5s_i^{k|M_{t-1,t}^{k}}}}{\prod_{i\in M_{t-1,k}}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5s_i^{t-1|M_{t-1,k}}} \prod_{i\in M_{t-1,k}}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5s_i^{k|M_{t-1,k}}}}$$

$$= \frac{\prod_{i\in M_{t-1,t}^{k}}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5s_i^{t-1|M_{t-1,t}^{k}}}}{\prod_{i\in M_{t-1,t}^{k}}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5s_i^{t-1|M_{t-1,k}}} \prod_{i\in D_{t-1,t}^{k}}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5s_i^{t-1|M_{t-1,k}}}} \tag{A.6}$$

$$\cdot \frac{\prod_{i\in M_{t-1,t}^{k}}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5s_i^{k|M_{t-1,t}^{k}}}}{\prod_{i\in M_{t-1,t}^{k}}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5s_i^{k|M_{t-1,k}}} \prod_{i\in D_{t-1,t}^{k}}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5s_i^{k|M_{t-1,k}}}}$$

Let us define the total share of $D_{t-1,t}^{k}$ with respect to the matched products $M_{t-1,k}$:

$$s_{D_{t-1,t}^{k}}^{t-1|M_{t-1,k}} = \frac{\sum_{i\in D_{t-1,t}^{k}} p_i^{t-1}q_i^{t-1}}{\sum_{i\in M_{t-1,k}} p_i^{t-1}q_i^{t-1}} \;, \tag{A.7}$$

$$s_{D_{t-1,t}^{k}}^{k|M_{t-1,k}} = \frac{\sum_{i\in D_{t-1,t}^{k}} p_i^{k}q_i^{k}}{\sum_{i\in M_{t-1,k}} p_i^{k}q_i^{k}} \;. \tag{A.8}$$

If $i \in M_{t-1,t}^{k}$, then we have:

$$s_i^{t-1|M_{t-1,k}} = s_i^{t-1|M_{t-1,t}^{k}} \cdot (1 - s_{D_{t-1,t}^{k}}^{t-1|M_{t-1,k}}) \;, \tag{A.9}$$

$$s_i^{k|M_{t-1,k}} = s_i^{k|M_{t-1,t}^{k}} \cdot (1 - s_{D_{t-1,t}^{k}}^{k|M_{t-1,k}}). \tag{A.10}$$

If $i \in D_{t-1,t}^{k}$, then we have:

$$s_i^{t-1|M_{t-1,k}} = s_i^{t-1|D_{t-1,t}^k} \cdot s_{D_{t-1,t}^k}^{t-1|M_{t-1,k}} \quad , \tag{A.11}$$

$$s_i^{k|M_{t-1,k}} = s_i^{k|D_{t-1,t}^k} \cdot s_{D_{t-1,t}^k}^{k|M_{t-1,k}} \quad . \tag{A.12}$$

Plugging A.9 – A.12 into A.6, we obtain[9]:

$$\begin{aligned}
&\frac{P_{t-1,k}^{M_{t-1,t}^k}}{P_{t-1,k}^{M_{t-1,k}}} = \\
&\frac{\prod_{i\in M_{t-1,t}^k}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5\left(s_i^{t-1|M_{t-1,t}^k}\cdot s_{D_{t-1,t}^k}^{t-1|M_{t-1,k}}\right)}\prod_{i\in M_{t-1,t}^k}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5\left(s_i^{k|M_{t-1,t}^k}\cdot s_{D_{t-1,t}^k}^{k|M_{t-1,k}}\right)}}{\prod_{i\in D_{t-1,t}^k}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5\left(s_i^{t-1|D_{t-1,t}^k}\cdot s_{D_{t-1,t}^k}^{t-1|M_{t-1,k}}\right)}\prod_{i\in D_{t-1,t}^k}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{0.5\left(s_i^{k|D_{t-1,t}^k}\cdot s_{D_{t-1,t}^k}^{k|M_{t-1,k}}\right)}} = \\
&\left(\frac{\prod_{i\in M_{t-1,t}^k}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{s_i^{t-1|M_{t-1,t}^k}}}{\prod_{i\in D_{t-1,t}^k}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{s_i^{t-1|D_{t-1,t}^k}}}\right)^{0.5s_{D_{t-1,t}^k}^{t-1|M_{t-1,k}}} \cdot \left(\frac{\prod_{i\in M_{t-1,t}^k}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{s_i^{k|M_{t-1,t}^k}}}{\prod_{i\in D_{t-1,t}^k}\left(\frac{p_i^k}{p_i^{t-1}}\right)^{s_i^{k|D_{t-1,t}^k}}}\right)^{0.5s_{D_{t-1,t}^k}^{k|M_{t-1,k}}} = \\
&\left(\frac{P_{t-1,k}^{GL|M_{t-1,t}^k}}{P_{t-1,k}^{GL|D_{t-1,t}^k}}\right)^{0.5s_{D_{t-1,t}^k}^{t-1|M_{t-1,k}}} \cdot \left(\frac{P_{t-1,k}^{GP|M_{t-1,t}^k}}{P_{t-1,k}^{GP|D_{t-1,t}^k}}\right)^{0.5s_{D_{t-1,t}^k}^{k|M_{t-1,k}}}
\end{aligned} \tag{A.13}$$

In a similar way, we have that:

$$\frac{P_{k,t}^{M_{t-1,t}^k}}{P_{k,t}^{M_{t-1,k}}} = \left(\frac{P_{k,t}^{GL|M_{t-1,t}^k}}{P_{k,t}^{GL|N_{t-1,t}^k}}\right)^{0.5s_{N_{t-1,t}^k}^{k|M_{k,t}}} \cdot \left(\frac{P_{k,t}^{GP|M_{t-1,t}^k}}{P_{k,t}^{GP|N_{t-1,t}^k}}\right)^{0.5s_{N_{t-1,t}^k}^{t|M_{k,t}}} . \tag{A.14}$$

[9] Although definitions differ slightly, this decomposition is consistent with the one presented in de Haan, J, & Krsinich, F (2014a) which compares the matched Törnqvist index with the imputation Törnqvist index.

Multiplying expressions A.13 and A.14 and averaging over the entire time window, one obtains the desired result.

Appendix 2:

Table 7 is an artificial example that shows the structure of the data received for digital cameras, with a subset of characteristics. Note that, for confidentiality reasons, sales and quantities have had random noise added and brand and model names are omitted completely.

Table 7. GfK consumer electronics scanner data structure

OBS #	PERIOD	QUANTITY SOLD	TOTAL SALES	MODEL	BRAND	CF CARD	CHIPTYPE	DIGITAL INDEX
1	Mar-11	7	2277.59	*	*	NO CF CARD	CCD	DIGITAL INDEX
2	Mar-11	388	146213.38	*	*	N.A.	CCD	DIGITAL INDEX
3	Mar-11	152	67078.15	*	*	N.A.	CCD	N.A.
4	Mar-11	143	88634.93	*	*	NO CF CARD	CCD	DIGITAL INDEX
5	Mar-11	132	80531.68	*	*	NO CF CARD	CCD	DIGITAL INDEX
6	Mar-11	103	109032.06	*	*	NO CF CARD	CMOS	DIGITAL INDEX
7	Mar-11	83	27314.77	*	*	NO CF CARD	CCD	DIGITAL INDEX
8	Mar-11	58	53096.64	*	*	NO CF CARD	CMOS	DIGITAL INDEX
9	Mar-11	43	40971.79	*	*	N.A.	CMOS	N.A.
10	Apr-11	43	29163.53	*	*	NO CF CARD	CCD	N.A.
11	Apr-11	37	43785.84	*	*	NO CF CARD	CMOS	DIGITAL INDEX
12	Apr-11	29	24746.86	*	*	NO CF CARD	CMOS	N.A.
13	Apr-11	27	11563.74	*	*	NO CF CARD	CCD	DIGITAL INDEX
14	Apr-11	25	10852.06	*	*	NO CF CARD	CCD	DIGITAL INDEX
15	Apr-11	23	17257.10	*	*	NO CF CARD	CMOS	DIGITAL INDEX
16	Apr-11	17	25483.29	*	*	NO CF CARD	CMOS	DIGITAL INDEX
17	Apr-11	14	13488.18	*	*	NO CF CARD	CMOS	N.A.
18	Apr-11	13	25640.43	*	*	NO CF CARD	CMOS	N.A.
19	Apr-11	17	18819.06	*	*	NO CF CARD	CMOS	DIGITAL INDEX
20	Apr-11	16	6821.91	*	*	NO CF CARD	CMOS	N.A.

OBS #	DIGITAL INPUT	HD Formats	IMAGE STABIL.	LCD SCREEN SIZE	MEMORY CAPACITY	OPTICAL ZOOM	OUTDOOR FUNCTIO	PIXEL TOTAL
1	NO DIG. INPUT	SD	ELEC.STAB	2.7	60	32	N.A.	1.07
2	NO DIG. INPUT	SD	OPTICAL STAB.	2.7	N.A.	70	NO WATER_SHOCK	0.8
3	DIGITAL INPUT	SD	ELEC.STAB	2.7	4	60	NO WATER_SHOCK	1.07
4	NO DIG. INPUT	SD	OPTICAL STAB.	2.7	80	78	N.A.	0.8
5	DIGITAL INPUT	SD	ELEC.STAB	2.7	80	60	NO WATER_SHOCK	0.68
6	NO DIG. INPUT	HD HDD	OPTICAL STAB.	2.7	120	25	NO WATER_SHOCK	3.32
7	DIGITAL INPUT	SD	ELEC.STAB	2.7	N.A.	39	NO WATER_SHOCK	0.8
8	DIGITAL INPUT	HD MEMORY	OPTICAL STAB.	2.7	N.A.	25	N.A.	3.32
9	DIGITAL INPUT	HD MEMORY	ELEC.STAB	2.7	8	25	N.A.	2.36
10	NO DIG. INPUT	SD	ELEC.STAB	2.7	16	37	N.A.	0.8
11	DIGITAL INPUT	HD HDD	ELEC.STAB	2.7	120	25	NO WATER_SHOCK	2.36
12	NO DIG. INPUT	HD MEMORY	ELEC.STAB	3	4	10	N.A.	10
13	NO DIG. INPUT	SD	ELEC.STAB	2.7	N.A.	52	NO WATER_SHOCK	0.8
14	NO DIG. INPUT	SD	ELEC.STAB	2.7	N.A.	37	NO WATER_SHOCK	0.8
15	NO DIG. INPUT	HD MEMORY	OPTICAL STAB.	2.7	N.A.	20	NO WATER_SHOCK	3.32
16	NO DIG. INPUT	HD MEMORY	OPTICAL STAB.	3	N.A.	12	N.A.	9.15
17	NO DIG. INPUT	HD MEMORY	ELEC.STAB	2.7	32	20	N.A.	2.39
18	NO DIG. INPUT	HD MEMORY	OPTICAL STAB.	3.5	64	10	NO WATER_SHOCK	6.63
19	DIGITAL INPUT	HD HDD	OPTICAL STAB.	2.7	160	12	NO WATER_SHOCK	7.1
20	NO DIG. INPUT	HD MEMORY	ELEC.STAB	2.7	8	30	NO WATER_SHOCK	3.32

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