Appendix 5 Calculation of the CPI by Laspeyres' Chain Index method

The weighted arithmetic mean with a fixed basket in the base period preceding the comparison period (Laspeyres formula) is used in many countries including Japan to calculate indices by combining the ratio of prices (indices) in the base period and the comparison period by using the weight of the base period. The Laspeyres formula is subdivided into the "fixed-base method" in which the index is calculated with the rate of consumption expenditure in the base year as the weight, and the "chain-linking method" in which the index of the current year, calculated based on the rate of consumption expenditure in the previous year as the weight, is multiplied every year.

Indices published in Japan are calculated by the fixed-base method, while those computed by the chain-linking method are also published as reference indices.

1 Basic formulas

In the chain-linking method, an index of a period is calculated by referring to the period immediately before that period (known as the "link index"), and the indices of two consecutive periods are multiplied in series to find the index (known as the "chain index").

The formulas used in this method are shown in 1) to 3). A link is made once a year, and the weight is revised every year using yearly average of the FIES (for households composed of two or more persons) in the previous year. The Laspeyres formula is used for finding the link indices.

1) Item and group indices excluding fresh food (e.g., "All items, less fresh food") (monthly indices)

The price index ratio by item used for the Laspeyres link index is calculated by dividing the price index of an item in the comparison period[61] by the price index of that item in December the previous year.

[61] The "Indices of items for Japan", calculated by the fixed-base method for each base, are used for price indices by item which are used for calculating the price index ratios by item.

<Group indices excluding fresh food (monthly indices)>

(Laspeyres link index (*L*))

$$I_{y,m}^{(L)} = \frac{\sum_{i=1}^{n} \frac{I_{y,m,i}}{I_{y-1,12,i}} w_{y-1,i}}{\sum_{i=1}^{n} w_{y-1,i}}$$

(Laspeyres chain index(*C*))

$$I_{y,m}^{(C)} = I_{0,12} \times \prod_{Y=1}^{y-1} I_{Y,12}^{(L)} \times I_{y,m}^{(L)}$$

Where, $I_{0,12}$ is the Laspeyres chain index in December 2020 with 2020 = 100[62]

(*Y*, *y*: Year, *m*: Month, 0: Base year, *i*: Item less fresh food, *n*: No. of items[63], *w*: Weight)

2) "Fresh fish & seafood," "Fresh vegetables", and "Fresh fruits" (monthly indices)

The group index ratios used for the Laspeyres link index are calculated by dividing the group index in the comparison period by the group index in December of the previous year.

<"Fresh fish & seafood," "Fresh vegetables", and "Fresh fruits" (monthly indices)>

(Laspeyres link index (*L*)) $I_{y,m,i}^{(L)} = \frac{I_{y,m,i}}{I_{y-1,12,i}}$

(Laspeyres chain index(*C*)) $I_{y,m,i}^{(C)} = I_{y-1,12,i}^{(C)} \times I_{y,m,i}^{(L)}$

(*y*: year, *m*: month, *i*: fresh fish & seafood, fresh vegetables, and fresh fruits)

3) Upper level group indices including fresh food (e.g., "All items" and "Food") (monthly indices)

Using the above formula showed in 1), the upper level group indices are calculated from the group indices excluding fresh food calculated in 1), and "Fresh fish & seafood," "Fresh vegetables", and "Fresh fruits" calculated in 2).

The monthly index is published in time with the publication of confirmed figures for the month in question, but for confirmed figures for January, a temporary index is calculated by using the weight two years ago due to incompletion of the previous year's weight. When the results of the FIES are published, a confirmed index is calculated with the previous year's weight and a revised index extending back to January is published in time with the publication of confirmed figures for February.

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[62] For each month of 2020, the Laspeyres link index is derived first with the 2015-base items and weight (2019 yearly averages). Then, the Laspeyres chain index in December 2020 is divided by the average of January to December 2020 to find the Laspeyres chain index for December 2020 ($I_{0,12}$) with 2020 = 100.

[63] It is $n = 1$ for the chain index of each item.

(Example of monthly index calculation)

To use a simple example, it is assumed that there is a subgroup "Meats" consisting of two items ("Beef" and "Pork"), and the price index for "beef" rises every year, and the price index for "pork" does not change. The weight of "beef" shrinks and the weight of "pork" expands as "beef" price rises.

Year and month	Price index*		Year	Weight	
	Beef	Pork		Beef	Pork
Dec., 00	100	100	00	10	10
Dec., 01	200	100	01	8	12
Jun., 02	400	100			

* Average in year 00 = 100

The index for "Meats" in June 02 calculated by the fixed-base method is given by averaging the price indices of "beef" and "pork" in June 02 with the weights of 00 as follows:

$$\frac{400 \times 10 + 100 \times 10}{10 + 10} = 250$$

To find the Laspeyres chain index for June 02, the Laspeyres link index for December of each year until the previous year (indices with the index for December of the previous year as 100) and Laspeyres link index for June 02 are calculated first.

The Laspeyres link index of December 01 is given by averaging the price index ratio of December 01 and December 00 with the weight of 00:

$$\frac{\frac{200}{100} \times 10 + \frac{100}{100} \times 10}{10 + 10} = 1.5$$

The Laspeyres link index of June 02 is given by averaging the price index ratio of June 02 and December 01 with the weight of 01:

$$\frac{\frac{400}{200} \times 8 + \frac{100}{100} \times 12}{8 + 12} = 1.4$$

Finally, the Laspeyres chain index of June 02 is calculated by multiplying the Laspeyres link indices of December 00 (100 is assumed here), December 01, and June 02:

$$100 \times 1.5 \times 1.4 = 210$$

2 Calculation of weights

(1) Weight reference period

The weight of each item for the Laspeyres link index is calculated from the annual average monthly expenditure by item per household in the previous year of the comparison period for that item, mainly obtained from the FIES for households consisting of two or more persons.

(2) Allocation rates

In calculating weights used for the fixed-base method, when more than one index item corresponds to one FIES item, the allocation rate is determined by the expenditure ratio which is obtained from the special tabulation of the FIES and other statistics. In calculating weights for the Laspeyres link index, the allocation rates of the base year (refer to "III Chapter 4, 2 Calculation of the weights for the basic classification indices") are used in general until the next base revision, due to restrictions in estimation data, but the allocation rates for items for which estimation data is available are revised as necessary.

(3) Weights for "Pocket money", etc.

In calculating weights used for the fixed-base method, the weights of "Pocket money" and "Social expenses" in the FIES are allocated based on the result of the NSFICW and their weights are calculated accordingly. As the data is obtained only once in five years, the allocation rate in calculating the weights for the Laspeyres link index is fixed until the next base revision.

(4) Weights for the imputed rent

In calculating weights used for the fixed-base method, the weights for "Imputed rent" are calculated for the base year according to the result of the NSFICW. The weights for the Laspeyres link index are given by multiplying the weight for "Imputed rent" in the base year by the increase-decrease rate of the price index of "Imputed rent" (for each municipality and other classifications) from the base year to the reference period.

3 Calculation of annual average index

The annual average index is the simple arithmetic average of monthly indices from January to December calculated by groups as described in Section 1.

4 Calculation of the rate of change

The same formula used in the fixed-base method is used. The rate of change is calculated with an index before the rounding of fractions. In the statistical tables, figures are rounded off to one decimal place.

5 Calculation of contribution

The contributions to month-to-month and year-ago month ratios of the all items index are given by the formulas shown below. The contribution to total change is calculated with an index before the rounding of fractions. In the statistical tables, figures are rounded off to two decimal places.

(1) Contribution to month-to-month ratio

$$\text{Contribution of item } i \text{ (February to December)}=\left[\frac{I^{(C)}_{y\text{-}1,12,\text{All items}}}{I^{(C)}_{y,m\text{-}1,\text{All items}}}\cdot\left(\frac{I^{(C)}_{y,m,i}-I^{(C)}_{y,m\text{-}1,i}}{I^{(C)}_{y\text{-}1,12,i}}\right)\cdot\frac{w_{y\text{-}1,i}}{w_{y\text{-}1,\text{All items}}}\right]\times 100$$

$$\text{Contribution of item } i \text{ (January)}=\left[\left(\frac{I^{(C)}_{y,1,i}-I^{(C)}_{y\text{-}1,12,i}}{I^{(C)}_{y\text{-}1,12,i}}\right)\cdot\frac{w_{y\text{-}1,i}}{w_{y\text{-}1,\text{All items}}}\right]\times 100$$

(Contribution: Contribution of item i to month-to-month ratio of all items index in month m of year y
$I^{(C)}_{y\,m\,i}$:Chain index of item i in month m of year y
$w_{y\,i}$:Weight of item i in year y n:All items)

(2) Contribution to year-ago month ratio

$$\text{Contribution of item } i=\frac{I^{(C)}_{y\text{-}2,12,\text{All items}}}{I^{(C)}_{y\text{-}1,m,\text{All items}}}\cdot\left(\frac{I^{(C)}_{y\text{-}1,12,i}-I^{(C)}_{y\text{-}1,m,i}}{I^{(C)}_{y\text{-}2,12,i}}\right)\cdot\frac{w_{y\text{-}2,i}}{w_{y\text{-}2,\text{All items}}}\times 100$$

$$+\frac{I^{(C)}_{y\text{-}1,12\text{All items}}}{I^{(C)}_{y\text{-}1,m,\text{All items}}}\cdot\left(\frac{I^{(C)}_{y,m,i}}{I^{(C)}_{y\text{-}1,12,i}}\text{-}1\right)\cdot\frac{w_{y\text{-}1,i}}{w_{y\text{-}1,\text{All items}}}\times 100$$

(Contribution: Contribution of item i to year-ago month ratio of all items index in month m of year y
$I^{(C)}_{y\,m\,i}$:Chain index of item i in month m of year y
$w_{y\,i}$:Weight of item i in year y)

6 Linking of old and new bases

Because the Laspeyres link index with the 2020-base items has been calculated since January 2021, the 2015-base Laspeyres chain index is calculated for up to December 2020, and a linked index having the 2020 average as 100 is calculated for up to December 2020.

The same procedure to link indices within a base period is used for linking the linked index and the 2020-base. The monthly Laspeyres chain index in 2021 is obtained by multiplying the Laspeyres chain index (linked index) for December 2020 calculated with 2015-base items by the relevant monthly Laspeyres link index in 2021 calculated with 2020-base items.

[Reference 1] History of index calculation using the chain-linking method

The indices obtained from the chain-linking method have been released as the reference annual average indices since 1975-base. The reference monthly indices for all items, less fresh food have been released from the 2005-base, and the reference monthly indices for all items including fresh food have been released from the 2015-base in addition to the calculation and publication of the contribution of each group and item.

[Reference 2] Characteristics of chain-linking method

Generally speaking, "the effect of weights", "effect of reset", and "drift phenomenon" are typical differences between the chain-linking method and fixed-base method.

With "the effect of weight", in addition to price fluctuation, the change in weights is reflected in the index and the rate of change in the chain-linking method, in which the weight is revised every year, if there is an item the rate of consumption of which increases or decreases with the fall or rise of price. However, the relations between price fluctuations and changes in the rate of consumption are not uniform, and the direction and significance largely differ depending on the characteristics of the item. It should, therefore, be noted that differences from the indices in the fixed-base method are not always seen in either the upper or lower parts.

"The effect of reset" causes differences from the fixed-base method in the contribution (degree of effect) of items because the indices level of items is reset every year in the chain-linking method, in which the index value of every item is calculated by assuming the index = 100 in December of previous year. If the price of an item has greatly dropped, and the price decline continues after the index value calculated by the fixed-base method has become considerably small, the range of drop would be greater in the chain-linking method than in the fixed-base method.

In the chain-linking method, the following phenomenon is pointed out: If the price of an item repeatedly rises and falls, the index values of the upper level group do not return to the normal level even if the item in question returns to the original price level. This is so-called the "drift phenomenon".

[Reference 3] "Link of price in December of previous year" and "link of average price of previous year"

In the chain-linking method, the chain index is calculated by multiplying the link indices of two consecutive periods in series, and there are two types of periods to link (linking points). These are, "link of the price in December of previous year" and "link of the average price of previous year".

While a "drift phenomenon" takes place in some cases in the chain-linking method, this phenomenon is seen more frequently in the "link of the price in December of previous year" than in the "link of the average price of previous year". At the same time, the "link of the average price of previous year" sometimes causes the indices of the upper level group to change due to the change of the linking point in December and January of the next year, even if the

price of the item in question does not change (this is called the occurrence of "discontinuation").

"Discontinuation" is considered undesirable in general, and for this reason, the "link of the price in December of previous year" is used for the items other than fresh food. The fresh food, however, contains seasonal items whose price repeatedly rises and falls in one-year cycle, causing a large-scale "drift phenomenon" when the "link of the price in December of previous year" is used, resulting in the indices of the upper level group to remain in the upper level even though the indices of the items in the same category return to the original level. That was the reason for using the "link of average price of previous year" exceptionally for fresh food in the 2015-base.

In the 2020-base, the calculation of a chain index of each item of fresh food is discontinued, and changes are made by handling the upper level group such as fresh fish & seafood as an item so that "discontinuation" and "drift phenomenon" do not affect the upper level group indices including fresh food, such as all items.

[Reference 4] Basics of decomposition of contribution for the chain-linking method

(1) Decomposition of the contribution for the year-ago month ratio in chain-linked indices

Similar to the technique used for the fixed-base method, the formula for the year-ago month ratio of the all items index for the chain-linked method is decomposed to the sum of rates of change for item i.

Considering the year-ago month ratio in the period from month m of year y - 1 to month m of year y, with linking point December of y - 1 as the boundary, the link index in the period from month m of year y - 1 to December of y - 1 is based on the weight of y - 2, while the link index in the period from December of year y - 1 to month m of year y is based on the weight of y - 1. Hence, the calculation of decomposition is separated to the contribution from month m of year y - 1 to December of year y - 1, and the contribution from December of year y - 1 to month m of year y. Of the formula for contribution to year-ago month ratio in 5 (2), the first term corresponds to the contribution from month m of year y - 1 to December of year y - 1, and the second term corresponds to the contribution from month 12 of year y - 1 to month m of year y.

However, it should be noted that according to this calculation formula, if the signs of the first and second terms are reversed, it may result in an inconsistent rate of change and contribution of items.

(2) Derivation of the formula for decomposing the contribution relating to the year-ago month ratio in chain-linked indices

Year-ago month ratio of the "All items" index in the Laspeyres chain index ($\times 100$ is omitted)

$$\frac{I^{(C)}_{y,m,\text{All items}} - I^{(C)}_{y-1,m,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}} = \frac{I^{(C)}_{y,m,\text{All items}} - I^{(C)}_{y-1,12,\text{All items}} + I^{(C)}_{y-1,12,\text{All items}} - I^{(C)}_{y-1,m,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}}$$

$$= \frac{I^{(C)}_{y-1,12,\text{All items}} I^{(L)}_{y,m,\text{All items}} - I^{(C)}_{y-1,12,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}} + \frac{I^{(C)}_{y-2,12,\text{All items}} I^{(L)}_{y-1,12,\text{All items}} - I^{(C)}_{y-2,12,\text{All items}} I^{(L)}_{y-1,m,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}}$$

$$= \frac{I^{(C)}_{y-1,12,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}} \left(I^{(L)}_{y,m,\text{All items}} - 1 \right) + \frac{I^{(C)}_{y-2,12,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}} \left(I^{(L)}_{y-1,12,\text{All items}} - I^{(L)}_{y-1,m,\text{All items}} \right)$$

$$= \frac{I^{(C)}_{y-1,12,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}} \left(\sum_i^n \frac{I^{(C)}_{y,m,i}}{I^{(C)}_{y-1,12,\text{i}}} \frac{w_{y-1,i}}{W_{y-1,\text{All items}}} - 1 \right) + \frac{I^{(C)}_{y-2,12,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}} \left(\sum_i^n \frac{I^{(C)}_{y-1,12,i}}{I^{(C)}_{y-2,12,i}} \frac{w_{y-2,i}}{W_{y-2,\text{All items}}} - \sum_i^n \frac{I^{(C)}_{y-1,m,i}}{I^{(C)}_{y-2,12,i}} \frac{w_{y-2,i}}{W_{y-2,\text{All items}}} \right)$$

$$= \frac{I^{(C)}_{y-1,12,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}} \left(\sum_i^n \left(\frac{I^{(C)}_{y,m,i}}{I^{(C)}_{y-1,12,\text{i}}} - 1 \right) \frac{w_{y-1,i}}{W_{y-1,\text{All items}}} \right) + \frac{I^{(C)}_{y-2,12,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}} \left(\sum_i^n \left(\frac{I^{(C)}_{y-1,12,i}}{I^{(C)}_{y-2,12,i}} - \frac{I^{(C)}_{y-1,m,i}}{I^{(C)}_{y-2,12,i}} \right) \frac{w_{y-2,i}}{W_{y-2,\text{All items}}} \right)$$

$$= \sum_i^n \left[\frac{I^{(C)}_{y-1,12,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}} \left(\frac{I^{(C)}_{y,m,i}}{I^{(C)}_{y-1,12,\text{i}}} - 1 \right) \frac{w_{y-1,i}}{W_{y-1,\text{All items}}} + \frac{I^{(C)}_{y-2,12,\text{All items}}}{I^{(C)}_{y-1,m,\text{All items}}} \left(\frac{I^{(C)}_{y-1,12,i}}{I^{(C)}_{y-2,12,i}} - \frac{I^{(C)}_{y-1,m,i}}{I^{(C)}_{y-2,12,i}} \right) \frac{w_{y-2,i}}{W_{y-2,\text{All items}}} \right]$$

Change from the previous month (January)

$$\frac{I^{(C)}_{y,1,\text{All items}} - I^{(C)}_{y-1,12,\text{All items}}}{I^{(C)}_{y-1,12,\text{All items}}} = \frac{I^{(C)}_{y-2,12,\text{All items}} I^{(L)}_{y-1,12,\text{All items}} I^{(L)}_{y,1,\text{All items}} - I^{(C)}_{y-2,12,\text{All items}} I^{(L)}_{y-1,12,\text{All items}}}{I^{(C)}_{y-2,12,\text{All items}} I^{(L)}_{y-1,12,\text{All items}}} = I^{(L)}_{y,1,\text{All items}} - 1$$

$$= \sum_i^n \frac{I^{(C)}_{y,1,i}}{I^{(C)}_{y-1,12,i}} \frac{w_{y-1,i}}{W_{y-1,\text{All items}}} - 1 = \sum_i^n \left(\frac{I^{(C)}_{y,1,i} - I^{(C)}_{y-1,12,i}}{I^{(C)}_{y-1,12,i}} \right) \frac{w_{y-1,i}}{W_{y-1,\text{All items}}}$$

Change from the previous month (February to December)

$$\frac{I^{(C)}_{y,m,\text{All items}} - I^{(C)}_{y,m-1,\text{All items}}}{I^{(C)}_{y,m-1,\text{All items}}} = \frac{I^{(C)}_{y-1,12,\text{All items}}}{I^{(C)}_{y,m-1,\text{All items}}} \left(I^{(L)}_{y,m,\text{All items}} - I^{(L)}_{y,m-1,\text{All items}} \right)$$

$$= \frac{I^{(C)}_{y-1,12,\text{All items}}}{I^{(C)}_{y,m-1,\text{All items}}} \left(\sum_i^n \frac{I^{(C)}_{y,m,i}}{I^{(C)}_{y-1,12,i}} \frac{w_{y-1,i}}{W_{y-1,\text{All items}}} - \sum_i^n \frac{I^{(C)}_{y,m-1,i}}{I^{(C)}_{y-1,12,i}} \frac{w_{y-1,i}}{W_{y-1,\text{All items}}} \right)$$

$$= \sum_i^n \frac{I^{(C)}_{y-1,12,All\ items}}{I^{(C)}_{y,m-1,All\ items}} \left(\frac{I^{(C)}_{y,m,i} - I^{(C)}_{y,m-1,i}}{I^{(C)}_{y-1,12,i}} \right) \frac{w_{y-1,i}}{W_{y-1,All\ items}}$$