On Measuring Regional or Global Growth and Inflation

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Outline

• Concepts for international comparison
  • Exchange rates (XR)
  • Purchasing power parities (PPPs)
• Comparisons of World GDP over time
  • Global Inflation
  • Global Growth
Key Concepts and Notation

Gross Domestic Product of country $j$
(in national currency units)

\[ GDP^j = \sum p^j_n \cdot x^j_n \]

Nominal GDP : GDP of country $j$ expressed in reference currency units using exchange rates

\[ NGDP^j = \frac{GDP^j}{XR^j} \]

Real GDP : GDP of country $j$ expressed in reference currency units using PPPs

\[ RGDP^j = \frac{GDP^j}{PPP^j} \]
Purchasing Power Parities (PPPs)

PPPs are amounts of currencies, of different countries, that have the same purchasing power as one unit of a reference currency (e.g. US$) with respect to a selected basket of goods and services (the scope). Methods were surveyed by Balk (2008), (2009).
Price Level Index (PLI)

Defined as the ratio of PPP to the exchange rate

\[ PLI^j = \frac{PPP^j}{XR^j} \]

PLIs are transitive, but not invariant to choice of reference country.
Normalisation of PLIs

Adjust the PPPs by a positive scalar such that world real GDP at the new PPPs is equal to world nominal GDP at XRs:

\[ \sum_j \frac{GDP_j}{PPP_j/\mu} = \sum_j \frac{GDP_j}{XR_j} \]

This represents current Eurostat National Accounting practice. PPPs are calculated according to the GEKS method.
Global Inflation and Growth (1)

Total real GDP in periods $s$ and $t$ (later) is given by

\[
RGDP^s = \sum_{j=1}^{M} RGDP_j^s = \sum_{j=1}^{M} GDP_j^s / PPP_j^s
\]

\[
RGDP^t = \sum_{j=1}^{M} RGDP_j^t = \sum_{j=1}^{M} GDP_j^t / PPP_j^t
\]

It is important to realize that these two aggregates are in the prices of periods $s$ and $t$ respectively. The ratio of these two aggregates is similar to the ratio of country-specific nominal GDP in two periods.
Global Inflation and Growth (2)

The objective is to decompose the total real GDP ratio into a price index and a quantity index,

\[
\frac{RGDP_t}{RGDP_s} = WP(s, t)WQ(s, t).
\]

Indices may be direct or chained. We prefer a symmetric (in time periods) to an asymmetric decomposition.
Global Inflation and Growth (3)

Using the logarithmic mean it appears that

\[
\frac{\sum_{j=1}^{M} GDP_j^t / PPP_j^t}{\sum_{j=1}^{M} GDP_j^s / PPP_j^s} = \exp \left\{ \sum_{j=1}^{M} \Psi_j \ln \left( \frac{GDP_j^t / PPP_j^t}{GDP_j^s / PPP_j^s} \right) \right\}
\]

\[
= \exp \left\{ \sum_{j=1}^{M} \Psi_j \ln \left( \frac{RGDP_j^t}{RGDP_j^s} \right) \right\}
\]

where the weights, adding up to 1, are defined by

\[
\Psi_j = \frac{L \left[ \frac{RGDP_j^s}{\sum_{k=1}^{M} RGDP_k^s} , \frac{RGDP_j^t}{\sum_{k=1}^{M} RGDP_k^t} \right]}{L(\ldots)}
\]

where \( L(a, b) = \frac{a - b}{\ln a - \ln b} \) is the logmean.
Global Inflation and Growth (4)

Using NA data country-specific nominal GDP change can be decomposed into price and quantity indices

\[
\frac{GDP^t_j}{GDP^s_j} = P_j^{GDP}(s,t) \cdot Q_j^{GDP}(s,t) \quad \forall j
\]

Then World Inflation and World Growth are measured by:

\[
WI(s,t) = \exp\left\{ \sum_{j=1}^{M} \Psi^j \ln\left( P_j^{GDP}(s,t) \cdot \frac{PPP^s_j}{PPP^t_j} \right) \right\}
\]

\[
WG(s,t) = \exp\left\{ \sum_{j=1}^{M} \Psi^j \ln\left( Q_j^{GDP}(s,t) \right) \right\}
\]
Global Inflation and Growth (5)

It appears that if the period $t$ PPPs are obtained by extrapolating the period $s$ PPPs then world inflation reduces to the price index of the numeraire country. Since there are $M$ choices for the numeraire, an unweighted geometric mean makes sense.
The entire derivation can be repeated for total nominal GDP. This delivers XR-based global price and quantity indices. They will differ numerically from the PPP-based indices. Different decompositions of the same ratio.

In both cases the global price index is not invariant to the choice of the reference country.

Instead of Sato-Vartia one could use Fisher.

Table 1 is based on ICP data: 2005, 2011, 141 countries.
### Table 1: Regional and Global Growth and Inflation, 2005 to 2011

<table>
<thead>
<tr>
<th>ICP Region</th>
<th>Exchange Rate Based</th>
<th>PPP Based</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NGDP 2011/NGDP 2005</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Price change (Fisher)</td>
<td>Price change (SV)</td>
</tr>
<tr>
<td>Asia and the Pacific</td>
<td>2.5297</td>
<td>1.5722</td>
</tr>
<tr>
<td>Africa</td>
<td>2.1443</td>
<td>1.4286</td>
</tr>
<tr>
<td>CIS</td>
<td>2.4622</td>
<td>1.9576</td>
</tr>
<tr>
<td>Eurostat-OECD</td>
<td>1.2888</td>
<td>1.2098</td>
</tr>
<tr>
<td>Latin America</td>
<td>2.5991</td>
<td>1.9725</td>
</tr>
<tr>
<td>Iran</td>
<td>2.7520</td>
<td>2.1138</td>
</tr>
<tr>
<td>West Asia</td>
<td>2.3175</td>
<td>1.5821</td>
</tr>
<tr>
<td>Georgia</td>
<td>2.108</td>
<td>1.6377</td>
</tr>
<tr>
<td>World</td>
<td>1.5388</td>
<td>1.3196</td>
</tr>
</tbody>
</table>


### Table 2: Components of Global Inflation

<table>
<thead>
<tr>
<th>ICP Region</th>
<th>Domestic Price Change 1</th>
<th>Exchange Rate Change</th>
<th>Domestic Price Change 2</th>
<th>PPP Change 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>World</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Why SV is preferred

• Simpler functional form.
• Decomposable (into 3 components; or contributions of groups of countries).
• See Table 2 and Table 3 (in paper).
Components of Inflation

Table 2: Components of Global Inflation

<table>
<thead>
<tr>
<th>ICP Region</th>
<th>Domestic Price Change&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Exchange Rate Change&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Domestic Price Change&lt;sup&gt;2&lt;/sup&gt;</th>
<th>PPP Change&lt;sup&gt;2&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asia and the Pacific</td>
<td>1.3945</td>
<td>1.1271</td>
<td>1.4354</td>
<td>1.0899</td>
</tr>
<tr>
<td>Africa</td>
<td>1.6089</td>
<td>0.8878</td>
<td>1.6257</td>
<td>0.8621</td>
</tr>
<tr>
<td>CIS</td>
<td>2.1326</td>
<td>0.9180</td>
<td>2.1478</td>
<td>0.9102</td>
</tr>
<tr>
<td>EuroStat-OECD</td>
<td>1.1104</td>
<td>1.0896</td>
<td>1.1244</td>
<td>1.0693</td>
</tr>
<tr>
<td>Latin America</td>
<td>1.6809</td>
<td>1.1736</td>
<td>1.6968</td>
<td>1.1557</td>
</tr>
<tr>
<td>Iran</td>
<td>2.5035</td>
<td>0.8444</td>
<td>2.5035</td>
<td>0.8444</td>
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<tr>
<td>West Asia</td>
<td>1.5661</td>
<td>1.0103</td>
<td>1.6155</td>
<td>0.9937</td>
</tr>
<tr>
<td>Georgia</td>
<td>1.5237</td>
<td>1.0748</td>
<td>1.5237</td>
<td>1.0748</td>
</tr>
</tbody>
</table>

| World              | 1.2148                           | 1.0861                          | 1.3058                           | 0.9464                 |

<sup>1</sup>Equation (21). <sup>2</sup>Equation (28).

whereby the Sato-Vartia indices possess the virtue of simple decomposability. The pair formed by the components of the Sato-Vartia indices decomposes the variances of the components of the Sato-Vartia indices.
Thank you!