Choice of index number formula and the upper-level substitution bias in the Canadian CPI

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1. Introduction

Statistics Canada currently applies the Lowe index formula, a widely used fixed–basket index, for the upper-level aggregates of its consumer price index (CPI), which measures the changes in the cost of purchasing a fixed basket of goods and services. If the main assumption of the index number theory—constant quality, is satisfied, the Lowe index reflects pure price changes between the two periods being compared. Applying this formula, a statistical agency can produce the unrevised CPI in a timely manner and easily interpret its meaning to the public.

There are, however, some inherent limitations associated with the Lowe index. It is commonly recognized that consumers' purchases, in terms of quantities, would generally vary with the changes in the relative prices of the goods and services. However, owing to the fixed–basket concept of the Lowe index, the quantity of the goods and services included in the basket must be unchanged within the life span of a CPI basket regardless of the existence of commodity substitution. Moreover, the lag between the basket reference period and its implementation time poses doubts on the representativeness of the expenditure pattern exhibited by the CPI basket. As a result, the Lowe index number formula is unable to fully account for the price-induced commodity substitution, and the resulting CPI is not a true measure of the actual change in the cost of living.

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The cost-of-living index (COLI) is another concept of CPI, which is based on economic theory. It measures the changes in the minimum cost of maintaining a given level of utility between the two comparison periods. The true cost-of-living index also takes into account changes in the governmental or environmental factors affecting consumers' well-being, which is generally difficult to measure. However, it is believed that the underlying COLI could be closely approximated by a small group of index number formulae belonging to the superlative indices that allow for substitutions among goods and services as relative prices change. The superlative indices are, therefore, recommended by the ILO CPI Manual[2] as the theoretical target index for the upper-level index.

In practice, it is not feasible to compute a timely CPI using a superlative index number formula because of the requirement of the information on consumers' expenditure patterns of the current period, which is normally not available at the time of index calculation. Nevertheless, retrospective superlative indices are often compiled by statistical agencies for estimating the upper-level commodity-substitution bias.[3]

Examining the sources of generating commodity-substitution bias, we believe that we could normally reduce it by increasing the frequency of CPI basket updates, by shortening the implementation lag of each new CPI basket, or by choosing an alternative index number formula for the upper-level aggregation. The recent existing literature associated with bias studies concentrates on the impact of the choice of index number formula on the CPI. For example, Hansen (2007) compared a Lowe index and a Young index, and found that the Lowe index generated higher upward bias in the CPI than the Young index using Danish data from 1996 to 2006. Lent and Dorfman (2009) conducted an empirical study and verified that a weighted average of the base-period price indices—arithmetic and geometric Laspeyres—could result in a close approximation to a superlative index using United States (U.S.) airfare data. Armknecht and Silver (2012) examined the use of geometric averaging—making use of the Geometric Young and Geometric Lowe index number formula—using U.S. CPI data. They also advocated

[2] The ILO CPI Manual is the short name used in this paper for the "Consumer Price Index Manual: Theory and Practice" (2004), which is jointly published by the International Labour Office, International Monetary Fund, Organization for Economic Co-operation and Development, United Nations and the World Bank.

[3] In this paper, the discussion is limited to the upper-level commodity-substitution bias.

using simple geometric means of one arithmetic average and one geometric average, for instance the geometric mean of the Lowe and Geometric Young, or the Young and Geometric Lowe, to approximate the superlative index.

As the empirical results of comparing the use of different index number formulae vary with a country's particular price trends during the comparison period and its associated consumers' expenditure patterns, we are not sure to what extent the conclusions drawn from an empirical study can be generalized from one country to another, and from one period to another, particularly whether these conclusions are also true for Canada in the current economy. Therefore, the empirical study using Canadian data is an important addition to the literature of bias study; and moreover, the mathematical analysis of the underlying reasons for choosing an index number formula would also provide us with the theoretical foundation on which to develop future strategies for improving the quality of the Canadian CPI.

In 2013, a more frequent CPI basket update schedule — from an update every four years to an update every two years—was implemented by Statistics Canada. Meanwhile, the 2011 basket was put into practice more quickly than in the past — from a lag of 16 months to a lag of 13 months. After these improvements, we investigate, with the same data required to compute a Lowe index, whether we can apply other strategies to reduce the commodity-substitution bias, and how different, in terms of the impact of reducing the substitution bias, the other strategies are compared with those already employed.

After having discussed the impact of the frequency of updating a CPI basket and the time lag of implementing a new basket on the substitution bias in our other paper—Huang, Wimalaratne and Pollard (2015), in this paper, we focus on examining the effect of applying different index number formulae on the upper-level commodity-substitution bias, using the Canadian data set,[4] constructed based on the Survey of Household Spending (SHS) and the CPI for the period from 2000 to 2013. Not only are the empirical results in the literature verified by using the Canadian data, but also some of the advocated index number formulae in the literature are modified and the corresponding numerical results are compared.

[4] For details on the construction of the data set, see Huang, Wimalaratne and Pollard (2015).

To proceed, we organize the rest of this paper as follows: section two briefly introduces the index number formula—the Lowe index, used to compile the official CPI in Canada; section three discusses the CPI series constructed using the symmetric index number formulae, including a group of superlative indices and some other symmetrically weighted indices; and section four examines the asymmetric index number formulae; the several other weighted price indices, using the same data required to construct the Canadian official CPI, are addressed in the subsequent section; finally, section six concludes the paper.

2. Official CPI—the Lowe Index

The calculation and release of a country's official CPI have to follow certain standards based on the main use of the CPI and data availability. The Canadian CPI is published monthly on the predetermined release dates; and the official CPI data are not revised once published, because they are widely used to adjust incomes, wages, and other payments.

To calculate price indices, we need information on both the prices and quantities. Usually, the information on prices can be obtained via an ongoing price survey, while the information on expenditure weights is derived from a country's household expenditure survey which is conducted infrequently. In Canada, a minimum of ten months is currently required to obtain and process necessary information for the CPI basket. Because of the data constraints and non-revisions policy, only certain index number formula, such as the Lowe and Young index number formulae can be implemented to compile the CPI in a timely manner.

The Lowe index number formula is used by most statistical agencies to construct their headline CPI. Statistics Canada compiles the CPI through two stages of aggregation, and uses the Lowe index number formula for the upper-level aggregation. In theory, a direct Lowe index $P_{Lo}(p^0, p^t, q^b)$ can be defined in terms of a quantity vector $q^b \equiv \left[q_{1,}^b \ldots q_{N,}^b \right]$, a price vector of base period $p^0 \equiv \left[p_{1,}^0 \ldots p_{N,}^0 \right]$ and a price vector of current period $p^t \equiv \left[p_{1,}^t \ldots p_{N,}^t \right]$:

(1) $$P_{Lo}(p^0, p^t, q^b) = \frac{\sum_{i=1}^{N} p_i^t \times q_i^b}{\sum_{i=1}^{N} p_i^0 \times q_i^b},$$

where N is the total number of goods and services included in the CPI basket.

In the second stages of the aggregation, the price vector and quantity vector are defined over the elementary aggregates, which are not observed directly in practice. To make it calculable, we can rewrite the above formula in terms of the hybrid-share form:

(2) $$\begin{aligned} P_{Lo}(p^0, p^t, s^{0:b}) &= \frac{\sum_{i=1}^{N} p_i^t \times q_i^b}{\sum_{i=1}^{N} p_i^0 \times q_i^b} = \sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0} \right) \frac{p_i^0 \times q_i^b}{\sum_{i=1}^{N} p_i^0 \times q_i^b} \\ &= \sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0} \right) s_i^{0:b} \end{aligned}$$

where

$\frac{p_i^t}{p_i^0}$ is the price index of elementary aggregate i between period *0* and t; and

$s_i^{0:b}$ is the hybrid expenditure share of elementary aggregate i corresponding to the quantity vector at weight reference year (b), q^b, measured at the price vector of the price reference period p^0. It is defined as:

(3) $$s_i^{0:b} = \frac{p_i^0 \times q_i^b}{\sum_{i=1}^{N} p_i^0 \times q_i^b}$$

In practice, the hybrid expenditures $p_i^0 \times q_i^b$ are obtained through a price-updating process:

(4) $$p_i^0 \times q_i^b = \left(\frac{p_i^0}{p_i^b} \right) \left(p_i^b \times q_i^b \right), \qquad i\text{=1, 2, …}N$$

In the direct Lowe index number formula, there are, therefore, three time periods involved in the index calculation: a weight reference period (b), a price reference period (*0*), and a current price observation period (t). In practice, the weight reference period is normally prior to the two price periods being compared. Since the weights are corresponding to neither periods being compared, the official CPI of Statistics Canada can be referred to as an asymmetrically weighted price index.

The upper-level substitution bias results not only from the representativeness of the expenditure patterns, which is associated with the CPI basket-update, but also from the choice of index number formula. In this paper, we focus on whether we could reduce the substitution bias by choosing an alternate index number formula. To do so, we first examine the symmetrically weighted price indices that allow for commodity-substitution, and then identify the target index for this study.

3. Symmetrically Weighted Price Indices

A symmetric index is the one that treats prices and quantities in both periods of price comparison in a symmetric manner, so that the index takes into account the changes in the expenditure patterns of consumers over the two price comparison periods. From the definition of the symmetric indices, it is known that the weight information required by the symmetric indices is different from those used in the official CPI series. We will examine how different symmetric indices behave and approximate each other.

3.1 Superlative Price Indices

The Fisher, Walsh and Törnqvist indices, also belonging to superlative indices, are three widely used symmetric indices. As superlative indices, they are flexible and expected to provide fairly close approximations to the underlying cost-of-living index. They are, therefore, recommended as theoretical target indices by the ILO CPI Manual.

The Fisher price index, P_F, is defined as the geometric mean of the based-period index—Laspeyres index, P_L, and the current-period index—Paasche index, P_P, that is:

(5) $$P_F^{t/0} = \left(P_L^{t/0} \times P_P^{t/0}\right)^{1/2} = \left(\frac{\sum_{i=1}^{N} p_i^t \times q_i^0}{\sum_{i=1}^{N} p_i^0 \times q_i^0} \times \frac{\sum_{i=1}^{N} p_i^t \times q_i^t}{\sum_{i=1}^{N} p_i^0 \times q_i^t} \right)^{1/2}$$

where $P_L^{t/0}$ is the Laspeyres index and $P_P^{t/0}$ is the Paasche index between period *0* and *t*.

The Walsh price index, P_W, is a basket index whose quantities are formed by a geometric average of quantities in the two periods compared. In this way, the Walsh price index reflects the pure price change over the two periods:

(6) $$P_W^{t/0} = \frac{\sum_{i=1}^{N} p_i^t \times \sqrt{q_i^t q_i^0}}{\sum_{i=1}^{N} p_i^0 \times \sqrt{q_i^t q_i^0}}$$

The Törnqvist price index, P_T, is defined as a geometric average of the price relatives weighted by the arithmetic average of the expenditure shares in the two comparison periods:

(7) $$P_T^{t/0} = \prod_{i=1}^{N} \left(\frac{p_i^t}{p_i^0} \right)^{\frac{1}{2}\left(s_i^0 + s_i^t\right)}$$

where

(8) $$s_i^0 = \frac{p_i^0 \times q_i^0}{\sum_{i=1}^{N} p_i^0 \times q_i^0} \quad \text{and} \quad s_i^t = \frac{p_i^t \times q_i^t}{\sum_{i=1}^{N} p_i^t \times q_i^t}$$

Since all the superlative indices require information on the quantities or expenditures of the current period *t*, it is impossible to apply them to compute a timely unrevised CPI, due to the time required to obtain the current period's quantity information. However, they can be compiled retrospectively. The theoretical benefits of the superlative indices lead them to be recommended as target indices by most economists and CPI compilers.

3.2 Other Symmetrically Weighted Price Indices

In addition to the superlative indices, there are several other symmetrically weighted price indices such as the Marshall-Edgeworth price index and the Drobisch price index.

The Marshall-Edgeworth index, P_{ME}, is also a basket index whose quantity weights are formed by an arithmetic average of the quantities in the two comparison periods. Like the Walsh index, it reflects the pure price movements over the two periods:

$$P_{ME}^{t/0} = \frac{\sum_{i=1}^{N} p_i^t \times \left[\left(q_i^0 + q_i^t\right)/2\right]}{\sum_{i=1}^{N} p_i^0 \times \left[\left(q_i^0 + q_i^t\right)/2\right]} \tag{9}$$

Similar to the Fisher price index, the Drobisch index, P_{DR}, is defined based on the Laspeyres and Paasche indices. However, it is the arithmetic mean of these two indices:

$$P_{DR}^{t/0} = \frac{1}{2}\left(P_L^{t/0} + P_P^{t/0}\right) = \frac{1}{2}\left(\frac{\sum_{i=1}^{N} p_i^t \times q_i^0}{\sum_{i=1}^{N} p_i^0 \times q_i^0} + \frac{\sum_{i=1}^{N} p_i^t \times q_i^t}{\sum_{i=1}^{N} p_i^0 \times q_i^t}\right) \tag{10}$$

Since the geometric mean is smaller than or equal to the arithmetic mean, the Drobisch price index should be greater than or equal to the corresponding Fisher index.

We can also produce an index constructed similarly to the Törnqvist index. Using the arithmetic mean instead of geometric mean, we can have the following Un-named index formula, P_{UN} :

$$\begin{aligned} P_{UN}^{t/0} &= \sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0}\right) \frac{1}{2}\left(s_i^0 + s_i^t\right) = \frac{1}{2}\sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0}\right) s_i^0 + \frac{1}{2}\sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0}\right) s_i^t \\ &= \frac{1}{2}\left(P_L^{t/0} + P_{Pal}^{t/0}\right) \end{aligned} \tag{11}$$

where the expenditure shares s_i^0 and s_i^t are defined in the same way as in equation (8). It is also a simple arithmetic mean of the Laspeyres index and the Palgrave index, which is an arithmetic mean of price relatives weighted by the current expenditure shares. Since this index does not satisfy the time reversal test and some other axiom tests, it does not have many attractions to both researchers and CPI compilers.

3.3 Relationship of the Symmetrically Weighted Price Indices

The different symmetrically weighted price indices mentioned in this section are compiled by using the constructed Canadian data set. As detailed monthly expenditure data are unavailable, only the annual price indices are compiled using the symmetrically weighted price index formulae with annual basket updates. Since multiple baskets are used in the calculation, the chain-linked price indices are calculated so as to link the price indices together across different CPI baskets.

Using the Fisher index number formula as an example, we illustrate how the chain-linked index between 2003 and 2011 is constructed:

$$P_{ChF}^{2011/2003} = \underbrace{P_F^{2004/2003}}_{P_F^{2004/2003}=\left(P_L^{2004/2003}\times P_P^{2004/2003}\right)^{1/2}} \times P_F^{2005/2004} \times \ldots \times P_F^{2010/2009} \times P_F^{2011/2010} \tag{12}$$

where $P_{ChF}^{2011/2003}$ denotes the chained Fisher index from 2003 to 2011; $P_F^{2004/2003}$ denotes the direct Fisher index from 2003 to 2004; $P_L^{2004/2003}$ denotes the direct Laspeyres index from 2003 to 2004, and $P_P^{2004/2003}$ denotes the direct Paasche index from 2003 to 2004. The other chain-linked symmetrically weighted price indices can be compiled similarly.

The following table shows the empirical results from the calculation.

Table 3.1: The chain-linked symmetrically weighted price indices (2003=100)

Year (t)	Superlative Indices			Other Symmetrically Weighted Indices		
	Fisher	Walsh	Törnqvist	Marshall-Edgeworth	Drobisch	Un-named
2003	100.000	100.000	100.000	100.000	100.000	100.000
2004	101.728	101.730	101.730	101.727	101.728	101.806
2005	103.746	103.750	103.750	103.744	103.746	103.919
2006	105.475	105.480	105.482	105.472	105.475	105.718
2007	107.401	107.409	107.410	107.398	107.401	107.716
2008	109.624	109.632	109.633	109.619	109.624	110.102
2009	109.670	109.684	109.688	109.663	109.670	110.393
2010	111.404	111.422	111.422	111.397	111.405	112.234
2011	114.389	114.408	114.405	114.381	114.390	115.374

For easy and clear comparison, we put the spread between the Fisher index and the other symmetrically weighted price indices in the following table. Through this table, we can clearly observe how the symmetrically weighted price indices differ from each other:

Table 3.2: The difference between the chained Fisher index and other symmetric price indices

Year (t)	Superlative Indices			Other Symmetrically Weighted Indices		
	Fisher	Walsh	Törnqvist	Marshall-Edgeworth	Drobisch	Unnamed
2003	0.000	0.000	0.000	0.000	0.000	0.000
2004	0.000	0.001	0.001	-0.001	0.000	0.078
2005	0.000	0.004	0.004	-0.002	0.000	0.173
2006	0.000	0.005	0.007	-0.004	0.000	0.243
2007	0.000	0.008	0.009	-0.004	0.000	0.315
2008	0.000	0.008	0.009	-0.005	0.000	0.478
2009	0.000	0.014	0.018	-0.007	0.000	0.723
2010	0.000	0.018	0.018	-0.008	0.000	0.830
2011	0.000	0.019	0.016	-0.008	0.000	0.984

The values in Table 3.2 represent the spread of index levels, in terms of percentage points, between the Fisher index and the other symmetrically weighted price indices. It shows that the spreads between the Fisher index and all the other symmetrically weighted price indices, except for the Un-named index, are fairly small. The results listed in Table 3.2 demonstrate that the most commonly used chained superlative indices generally closely approximate each other. During the periods examined, both the Walsh index and Törnqvist index are slightly higher than the Fisher index. The minor spreads between the Fisher index and Drobisch index imply that the differences between the Laspeyres and Paasche indices are not significant enough to cause big gaps between the Fisher and Drobisch for the examined periods. Moreover, among all the symmetrically weighted price indices the Marshall-Edgeworth index yields the lowest values in most periods and the Un-name index generates the highest values. The significant divergence between the Un-name index and the Fisher index exhibited in Table 3.2 implies that not all the symmetrically weighted price indices approximate each other and generate ideal results.

The symmetrically weighted price indices treat the quantities and prices in the two periods under the consideration equally, which allows for commodity substitution caused by the price changes. In addition, the superlative indices have a close connection to economic theory. Their theoretical attractions make them outweigh the other symmetric indices and their asymmetric counterparts. However, it is impossible to employ these indices in a real CPI in practice due to the lack of necessary data.

Following the recommendation made by the ILO CPI Manual, we choose one of the widely used superlative indices—the chained Fisher index as defined by equation (10), as the target index in this study.[5] It is used to determine how well the other index number formulae can track this target index.

4 Asymmetrically Weighted Price Indices

An asymmetrically weighted index is one where weights used to aggregate the elementary price indices are not associated with both periods being compared. In this manner, an asymmetrically weighted price index cannot reflect changes in the consumers' expenditure patterns across the

[5] This does not imply that Statistics Canada choose the Fisher index as the Target index.

two periods of price comparison, and is therefore subject to either upward or downward bias. The most typical asymmetrically weighted price indices are Laspeyres and Paasche indices.

In this section, we investigate the possibility of using asymmetrically weighted price indices other than the Lowe index to better track the target index. We start the discussion with the base-period price index—the Laspeyres index, which is a specific form of the Lowe index where the quantity weights are derived from the information corresponding to the price reference period. The direct Laspeyres index, $P_L^{t/0}$, can be defined as follows:

$$P_L^{t/0} = \frac{\sum_{i=1}^{N} p_i^t q_i^0}{\sum_{i=1}^{N} p_i^0 q_i^0} \qquad (13)$$

Using the strategy illustrated in the ILO CPI Manual,[6] we can compare the Lowe index with the Laspeyres. For the Lowe index, the basket reference period (*b*) is different from the two price comparison periods. More specifically, to produce the non-revised CPI in a timely manner the basket reference period (*b*) should be prior to the price reference period (*0*) for the Lowe index because of the timeline of the household expenditure survey. Decomposing the difference between the Lowe index and the Laspeyres index, we have the following expression:

$$\begin{aligned} P_L^{t/0} - P_{Lo}^{t/0} &= \frac{\sum_{i=1}^{N} p_i^t q_i^0}{\sum_{i=1}^{N} p_i^0 q_i^0} - \frac{\sum_{i=1}^{N} p_i^t q_i^b}{\sum_{i=1}^{N} p_i^0 q_i^b} \\ &= \sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0} - P_L^{t/0} \right) \left(\frac{q_i^0}{q_i^b} - Q_P^{0/b} \right) \frac{s_i^{0:b}}{Q_P^{0/b}} \end{aligned} \qquad (14)$$

[6] Refer to Appendix 15.2 of the ILO CPI Manual.

where:

$Q_P^{0/b}$ is the Paasche quantity index between the weight reference period (*b*) and the price reference period (*0*), which is defined as

(15) $$Q_P^{0/b} = \frac{\sum_{i=1}^{N} p_i^0 q_i^0}{\sum_{i=1}^{N} p_i^0 q_i^b}.$$

$s_i^{0:b}$, defined by equation (3), is the hybrid expenditure shares corresponding to the quantity weights vector q^b measured at the price vector of the price reference period p^0.

The first line of equation (14) indicates that the Laspeyres index employs more up-to-date weights (q^0) than the Lowe index. Because of price-induced commodity-substitution, we would expect a higher Lowe index series in the general case. The second line of equation (14) indicates that the covariance between the deviation of relative prices, $\left(\frac{p_i^t}{p_i^0} - P_L^{t/0}\right)$, and the deviation of relative quantities, $\left(\frac{q_i^0}{q_i^b} - Q_P^{0/b}\right)$, are for different time periods. This implies that if price trends persist from period *b* to *0* and continue on from period *0* to *t*, and price-induced substitution behaviour exists as expected, the Lowe index is likely to be higher than the Laspeyres index.

Different from the Laspeyres index, the Paasche index, $P_P^{t/0}$, is defined based on the current quantity weights as follows, which can also be written as a harmonic mean of the price relatives weighted by current expenditure shares:

(16) $$P_P^{t/0} = \frac{\sum_{i=1}^{N} p_i^t q_i^t}{\sum_{i=1}^{N} p_i^0 q_i^t} = \left[\sum_{i=1}^{N} s_i^t \left(\frac{p_i^t}{p_i^0}\right)^{-1}\right]^{-1}$$

Similarly, we can decompose the difference between the Paasche index and the Lowe index as follows:

(17) $$P_P^{t/0} - P_{Lo}^{t/0} = \frac{\sum_{i=1}^{N} p_i^t q_i^t}{\sum_{i=1}^{N} p_i^0 q_i^t} - \frac{\sum_{i=1}^{N} p_i^t q_i^b}{\sum_{i=1}^{N} p_i^0 q_i^b}$$
$$= \sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0} - P_L^{t/0} \right) \left(\frac{q_i^t}{q_i^b} - Q_{Lo}^{t/b} \right) \frac{s_i^{0:b}}{Q_{Lo}^{t/b}}$$

where:

$Q_{Lo}^{t/b}$ is the Lowe quantity index between the weight reference period (*b*) and the current period (*t*), measured at the prices of the price reference period, p_i^0. It is defined as:

(18) $$Q_{Lo}^{t/b} = \frac{\sum_{i=1}^{N} p_i^0 q_i^t}{\sum_{i=1}^{N} p_i^0 q_i^b}$$

From this decomposition, it can be seen that the Paasche price index is likely to be less than the Lowe index as long as the price trend between price reference period *0* and current period *t* is in the same direction as the price trend between basket reference period *b* and current period *t*.

Another price index where the weights are also derived from the current expenditure pattern is the Palgrave price index, $P_{Pal}^{t/0}$, which is defined based on current expenditure shares as follows:

(19) $$P_{Pal}^{t/0} = \sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0} \right) s_i^t$$

where

s_i^t is the current-period expenditure share, defined as:

(20) $$s_i^t = \frac{p_i^t q_i^t}{\sum_{i=1}^{N} p_i^t q_i^t}$$

The Palgrave price index is an arithmetic weighted average of the price relatives in the basket. The weights are expenditure shares derived from the current expenditure patterns of consumers, and vary with the change of the current period (t).

The Laspeyres index can be also written in terms of the expenditure share as follows:

(21) $$P_L^{t/0} = \frac{\sum_{i=1}^{N} p_i^t q_i^0}{\sum_{i=1}^{N} p_i^0 q_i^0} = \sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0} \right) s_i^0$$

where:

s_i^0 are the base-period expenditure shares, defined as:

(22) $$s_i^0 = \frac{p_i^0 q_i^0}{\sum_{i=1}^{N} p_i^0 q_i^0}$$

Different from the Palgrave price index, the weights for the Laspeyres index are fixed as long as the price reference period (*0*) is unchanged.

Price index differences between the Lowe index and Palgrave index can be decomposed as follows:

(23) $$\begin{aligned} P_{Pal}^{t/0} - P_{Lo}^{t/0} &= \sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0} \right) s_i^t - \frac{\sum_{i=1}^{N} p_i^t q_i^b}{\sum_{i=1}^{N} p_i^0 q_i^b} = \sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0} \right) s_i^t - \sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0} \right) s_i^{0:b} \\ &= \sum_{i=1}^{N} \left(\frac{p_i^t}{p_i^0} - P_{Lo}^{t/0} \right) \left(s_i^t - s_i^{0:b} \right) \end{aligned}$$

In equation (23) the covariance term is between the deviation of price relatives from their mean, $\left(\frac{p_i^t}{p_i^0} - P_{Lo}^{t/0} \right)$, and the difference in expenditure shares pertaining to current period *t* and the hybrid expenditure shares $\left(s_i^t - s_i^{0:b} \right)$. It is difficult to answer whether the expenditure share goes up or down as the price of item *i* goes up. The answer to this question depends on the elasticity of demand for the product as well as the price trends.

In Table 4.1 below, we show the different chained CPI series calculated using the Lowe index and the three asymmetrically weighted price indices in comparison to the Fisher index that is used as the target index in this study.

Table 4.1: The chain-linked asymmetrically weighted price indices

	Target Index	Canadian Official Index	Asymmetrically Weighted Price Indices		
	Fisher	Lowe	Laspeyres	Paasche	Palgrave
2003	100.000	100.000	100.000	100.000	100.000
2004	101.743	101.916	101.802	101.684	101.840
2005	103.768	104.176	103.925	103.611	103.955
2006	105.503	106.204	105.712	105.295	105.777
2007	107.445	108.310	107.687	107.202	107.826
2008	109.694	110.813	110.068	109.320	110.267
2009	109.753	111.095	110.343	109.166	110.601
2010	111.503	112.921	112.140	110.870	112.510
2011	114.466	115.987	115.123	113.813	115.759

The chained Fisher CPI series is highlighted in green and the chained Lowe CPI series is highlighted in purple. By looking at the price indices of each period in Table 4.1, it can be seen that the Lowe index number formula yields the highest index values. This indicates the presence of persistent price trends in the Canadian economy. In addition, the higher chained Palgrave CPI series relative to the chained Laspeyres series illustrates the non-proportional growth of prices and quantities in the economy, in another word, the expenditure shares, on average, move in the same direction as the changes in the price relatives.

Table 4.2 shows the annual inflation rates calculated based on the different CPI series.

Table 4.2: Annual growth rates of the different CPI series

	Target Index	Official Index	Asymmetrically Weighted Indices		
	Fisher	Lowe	Laspeyres	Paasche	Palgrave
2003	0.000	0.000	0.000	0.000	0.000
2004	1.743	1.916	1.802	1.684	1.840
2005	1.990	2.217	2.086	1.895	2.077
2006	1.672	1.947	1.719	1.625	1.752
2007	1.840	1.983	1.869	1.811	1.937
2008	2.093	2.311	2.211	1.975	2.264
2009	0.054	0.254	0.250	-0.141	0.303
2010	1.595	1.644	1.628	1.561	1.726
2011	2.657	2.715	2.660	2.655	2.888
Average Growth Rate (2003-2011)	1.703	1.871	1.776	1.630	1.846

A quick glance at the annual growth rates in the CPI series shows that the Lowe CPI series has the highest inflation rates for the period from 2003 to 2011, which also implies the highest substitution bias. Among the two current-period price indices, the Paasche CPI series that uses quantity weights yields lower inflation rate than the Palgrave CPI series that uses expenditure-share weights, which is not surprising since a weighted harmonic mean is lower than the corresponding arithmetic mean. Even though all the chained asymmetrically weighted price indices listed in Table 4.2 can improve the performance of the official index in terms of reducing the substitution bias, they cannot be applied to produce a timely CPI series for a variety of reasons including the non-revisions policy and the lack of timely information on current weights.

In the next section, we will discuss a group of index number formulae where the weights are derived from the same information used for calculating the Lowe index.

5 Other Weighted Price Indices

As previously mentioned, for a variety of reasons it is not feasible to publish the symmetrically weighted and asymmetrically weighted price indices examined in previous sections. However, with the same data required to calculate the Lowe index, it is still possible to identify some alternative price index formulae to compile the CPI under the operating constraints.

In this section, we will explore this possibility and examine the relationship between the upper-level substitution bias and the choice of the index number formula. To proceed, we first calculate the CPI series by applying different index number formulae, such as the Young index, the Geometric Lowe index, the Geometric Young index and the geometric mean of the arithmetic weighted and geometric weighted indices, etc., and then compare these CPI series with both the target index and the Lowe index.

5.1 The Young Index

The Young index, another popular choice of statistical agencies for compiling their CPIs at the upper-level aggregation, is defined as follows:

(24) $$P_Y(p^0, p^t, s^b) = \sum_i s_i^b \left(\frac{p_i^t}{p_i^0} \right),$$

where s_i^b is the expenditure share at the basket reference period b, defined as:

(25) $$s_i^b = \frac{p_i^b q_i^b}{\sum_i p_i^b q_i^b}$$

In this formula, the basket reference period (*b*) is prior to the price reference period (*0*).

Comparing the Young index with the Lowe index defined by equation (1), we can see that while the CPI basket for both indices is fixed for a certain time period, the Lowe index fixes the quantity weights while the Young index fixes the expenditure shares, which also means that the price-updating process from basket reference period (*b*) to price reference period (*0*) is necessary only for the Lowe index. We can decompose the difference between the Lowe index and the Young index in the following way:

$$
\begin{aligned}
(26)\quad & P_{Lo}(p^0, p^t, s^{0,b}) - P_Y(p^0, p^t, s^b) = \sum_{i=1}^{N} \frac{p_i^t}{p_i^0} s_i^{0,b} - \sum_i \frac{p_i^t}{p_i^0} s_i^b \\
& = \sum_i \left[\frac{p_i^0}{p_i^b} - P_Y(p^b, p^0, s^b) \right] \times \left[\frac{p_i^t}{p_i^0} - P_Y(p^0, p^t, s^b) \right] \times \left(\frac{s_i^b}{P_Y(p^b, p^0, s^b)} \right)
\end{aligned}
$$

The last line of the equation (26) shows a kind of correlation between the two price relatives above or below their Young means, where the first set of price deviations are from the basket reference period (*b*) to the price reference period (*0*), and the second set of deviations are from the price reference period (*0*) to the current period (*t*). If the price trends from basket reference period (*b*) to price reference period (*0*) and from price reference period (*0*) to current price period (*t*) are in the same direction, the above correlation will be positive and the Lowe index will likely to exceed the Young index. The ILO CPI Manual explains this point intuitively:

> "[If] *there are long-term trends in the prices, so that prices which have increased relatively from b to 0 continues to do so from 0 to t, and prices which have fallen from b to 0 continues to fall, the Lowe index will exceed the Young index."*[7]

In this decomposition of index difference, the relationship between the two indices is independent of changes in the expenditure pattern. In another word, consumers' substitution responses to price changes are not necessary for determining the difference between these two indices.

The following chart shows the comparison between the Lowe index and the Young index from January 2002 to December 2013. The CPI basket is updated every 2 years, and the time lag of introducing a new CPI basket is 13 months.[8] A higher Lowe index series than the corresponding Young index series in most of the time under examination is exhibited in Figure 5.1, which

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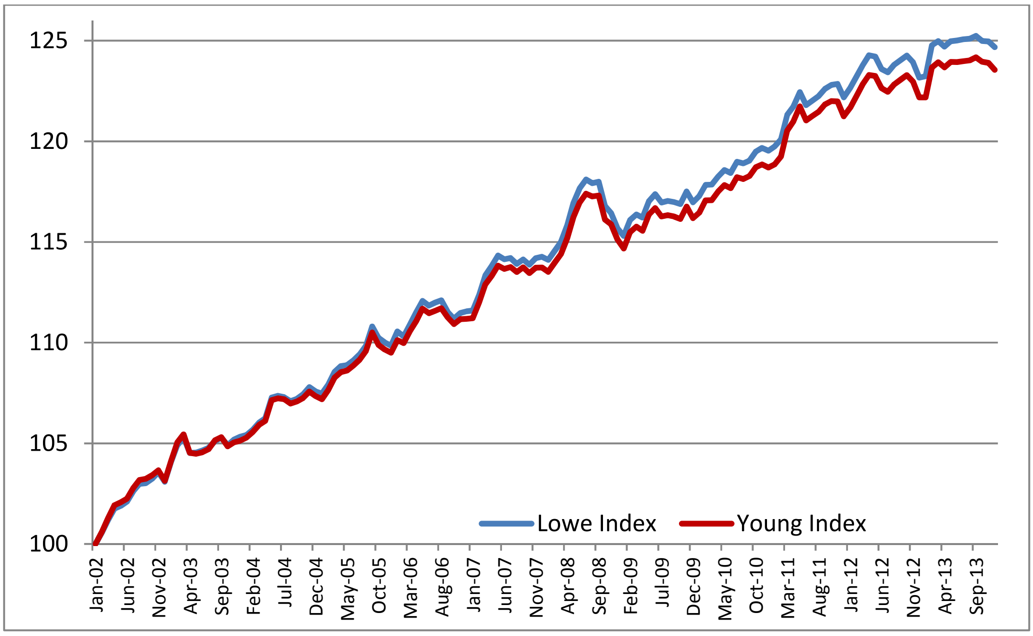
[7] Refer to the revised Chapter 9 Paragraph 9.115 of the ILO CPI Manual.

[8] This is same as the current Canadian practice.

implies the existence of persistent long-term price trends in the Canadian economy. This finding is consistent with most of the empirical results in the existing literature.

Figure 5.1: Comparison between the Lowe index and the Young index (January 2002-December 2013)



We are also interested in knowing how the spread between the Lowe index and the Young index changes with the frequency of basket updates and the implementation lag. The following table shows the difference between the Lowe index and the Young index with different basket-update frequencies and implementation lags:

Table 5.1: The differences between the Lowe index and the Young index

	Chained Lowe index - Chained Young index ($P_{ChLo} - P_{ChY}$)		
	(basket update every 2 years, lag of 13months)	(basket update each year, lag of13months)	(basket update each year, lag of 12months)
2002	-0.125	-0.125	-0.125
2003	0.015	-0.028	-0.049
2004	0.147	0.145	0.062
2005	0.293	0.318	0.147
2006	0.358	0.378	0.298
2007	0.444	0.357	0.350

2008	0.638	0.510	0.521
2009	0.687	0.291	0.158
2010	0.778	0.236	0.057
2011	0.805	0.370	0.200
2012	0.969	0.511	0.313
2013	1.066	0.533	0.319

The positive differences between the chained Lowe indices and the chained Young indices in Table 5.1 indicate that the Lowe index is higher than the Young index in most periods. Comparing the column blue and yellow, we can see that the differences between the Lowe indices and the Young indices are reduced when we accelerate the frequency of basket updates and shorten the time lag of introducing the new baskets, which also implies that, as the basket updates occur more frequently, and the new baskets are implemented more quickly, choosing either the Lowe index or the Young index provides diminishing gains in reducing the substitution bias. This effect is more pronounced when considering the long-term accumulative difference between the two indices.

5.2 Geometric Price Indices

In addition to the weighted arithmetic average of price relatives, using the same data as required to calculate the Lowe index, we can also produce corresponding geometric counterparts on a timely basis. The ILO CPI Manual suggests that these geometric indices are likely to be less subject to the price-induced substitution bias than their arithmetic counterparts. Therefore, "*these geometric indices must be treated as serious practical possibilities for purposes of CPI calculations*".[9]

[9] Refer to Chapter 1, paragraph 1.40 of the ILO CPI Manual.

The geometric version of the Lowe index[10] is defined as follows:

$$(27) \qquad P_{GLo}(p^0, p^t, s^{0:b}) = \prod_{i=1} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^{0:b}}$$

where $s_i^{0:b} = \dfrac{p_i^0 \times q_i^b}{\sum_i p_i^0 \times q_i^b}$ is the hybrid expenditure share.

The Geometric Young index is defined as:

$$(28) \qquad P_{GY}(p^0, p^t, s^b) = \prod_{i=1} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^b},$$

where $s_i^b = \dfrac{p_i^b q_i^b}{\sum_i p_i^b q_i^b}$ is the original expenditure share obtained at the weight reference period (b).

Armknecht and Silver (2012) verified that the long-run price changes from the weight reference period (*b*) to the current period (*t*) determine whether the Geometric Lowe exceeds the Geometric Young or vice-versa. The decomposition of the index difference between the Lowe and the Young indices in equation (26) indicates that it is the same long-run price trends determine the relationship between the Lowe index and the Young index. We would therefore expect the same relationship between the two sets of price indices.

The following chart shows different CPI series calculated using the weighted arithmetic average—the Lowe and the Young index, and their geometric counterparts. The CPI basket is assumed to be updated every 2 years and implemented 13 months after the basket reference period.

10 Paragraph 1.38 of the ILO CPI Manual points out "there are no counterpart Lowe indices", because it is difficult to justify the hybrid expenditure share weights. In this study, we use this type of formula to make the mathematic comparison.

Figure 5.2: Weighted geometric index and weighted arithmetic index (January 2002-December 2013)

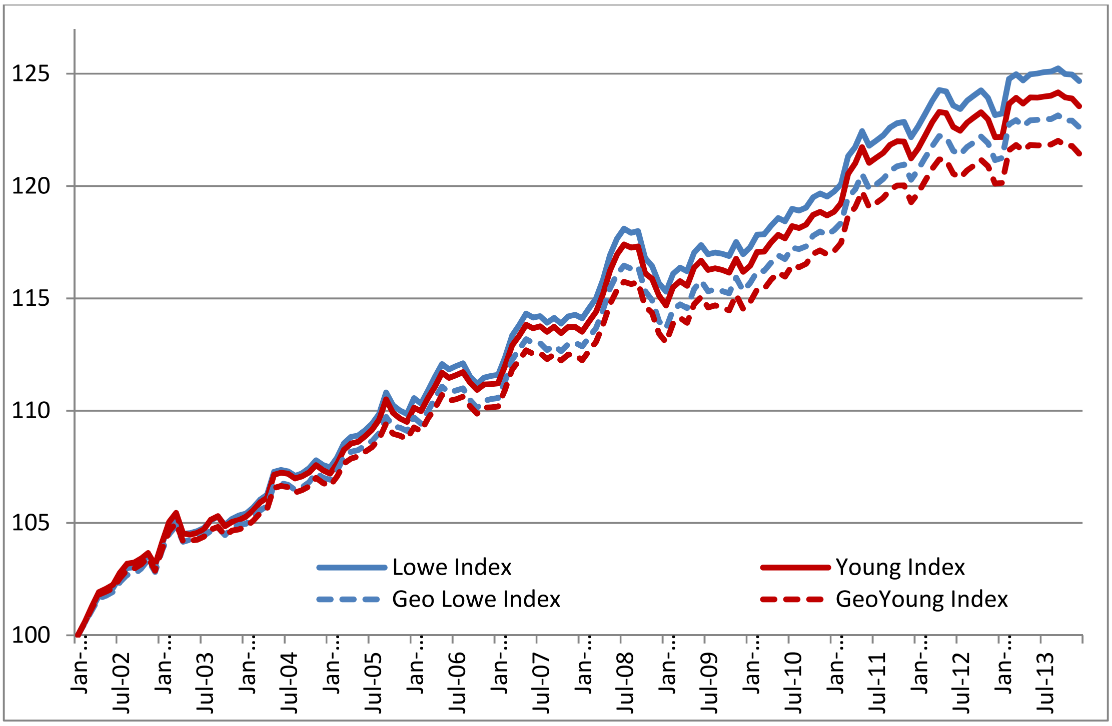


Figure 5.2 illustrates that the two geometric indices are lower than their arithmetic counterparts. For most months, the Geometric Young series lies below the Geometric Lowe series, similar to the relationship between the Young and Lowe series. This implies that most Canadian goods and services have experienced long-term persistent price trends in the examined period.

5.3 Other Weighted Price Indices

The Lloyd-Moulton index number formula defined by equation (29), which can be potentially used to produce the timely CPI, is also based on base-period expenditure shares.

$$(29) \qquad P_{LM}^{t/0} = \left\{ \sum_{i=1}^{N} s_i^0 \left(\frac{p_i^t}{p_i^0} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}$$

where:

σ is the elasticity of substitution[11] between the commodities covered in the aggregate.

The resulting CPI will be free of substitution bias to a certain degree. However, more effort is required to estimate the elasticity of substitution (σ), and the estimation itself is both subjective and non-reproducible. Moreover, similar to the Laspeyres index, we cannot obtain the base-period expenditure share estimates quickly enough due to the operation constraints of the household expenditure survey. To make it useful in the practice, we can modify the Lloyd-Moulton index slightly as follows:

$$(30) \qquad P_{ModLM}^{t/0} = \left\{ \sum_{i=1}^{N} s_i^b \left(\frac{p_i^t}{p_i^0} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}$$

where $s_i^b = \frac{p_i^b q_i^b}{\sum_{i=1}^{N} p_i^b q_i^b}$ is the expenditure share at the basket reference period (b) that is prior to the price reference period (0), and the elasticity of substitution[12] (σ) can be estimated using historical data.

In addition, Lent and Dorfman (2009) found that a weighted average of the base-period price indices (the Laspeyres) and its geometric counterpart (the Geometric Laspeyres), called the arithmetic AG Mean index (LD index in this study) and defined as follows, can approximate a superlative target index quite well:

$$(31) \qquad P_{LD}(p^0, p^t, s^0) = \sigma \prod_{i=1} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^0} + (1-\sigma) \sum_{i=1} \left(\frac{p_i^t}{p_i^0} \right) s_i^0$$

where the weight σ is the elasticity of substitution in the Lloyd-Moulton index. It is not restricted to be consistent over time. Since the base-period expenditure shares are available only after a certain time lag, the LD index cannot be used to compile the CPI on a timely basis. To enable its

[11] If σ=0, the Lloyd-Moulton index reduces to the Laspeyres index.

[12] If σ=0, the modified Lloyd-Moulton index (equation 26) reduces to the Young index.

application in the CPI in practice, we can alter this index by using expenditure shares at basket reference period (*b*). The transformed index is as follows:

$$(32) \qquad P_{ModLD}(p^0, p^t, s^0) = \sigma \prod_{i=1} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^b} + (1-\sigma) \sum_{i=1} \left(\frac{p_i^t}{p_i^0} \right) s_i^b$$

The modified LD index[13] is actually the weighted arithmetic mean of the Geometric Young and Young indices.

Maintaining the accuracy of the estimate of the elasticity of substitution between the products is crucial for determining how well both the modified Lloyd-Moulton and the modified LD indices compare to a superlative index. Using the constructed data from 2000 to 2011, we estimate the elasticity of substitution by tracking the three superlative indices as closely as possible. We find that the elasticity estimates are quite close to those obtained by using the method proposed by Lent and Dorfman (2009). The following table shows the elasticity estimated while tracking the different superlative indices. It demonstrates that the yearly estimates are unstable over the examination period. As a result, the average value of the estimates over the eleven years is used as the historical estimate in the calculation.

Table 5.2: The estimates of the elasticity of substitution between the commodities

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Average
σ—Fisher	0.495	0.740	0.444	0.727	1.023	0.862	0.538	0.840	0.882	0.409	0.350	0.665
σ—Walsh	0.506	0.729	0.448	0.709	0.997	0.844	0.499	0.837	0.859	0.370	0.347	0.650
σ—Törnqvist	0.510	0.727	0.455	0.709	0.992	0.821	0.510	0.841	0.845	0.415	0.367	0.654

The modified Lloyd-Moulton index is calculated using the above elasticity estimates to assess the sensitivity of the index value with respect to different estimates of elasticity. Some of the calculated index values and their associated geometric average growth rates are reported in Table 5.3:

[13] Same as the modified Lloyd-Moulton index, if σ=0, the modified LD index (equation 31) reduces to the Young index.

Table 5.3: The modified Lloyd-Moulton index series corresponding to different estimates of elasticity of substitution:

σ	Index Values (2003-2011)	Average Annual Growth Rate	Difference in Growth Rate
Fisher	114.389	1.695	0.000
0.347	114.981	1.760	0.066
0.409	114.891	1.750	0.056
0.455	114.822	1.743	0.048
0.499	114.758	1.736	0.041
0.538	114.700	1.729	0.035
0.727	114.422	1.698	0.004
0.740	114.402	1.696	0.001
0.821	114.283	1.683	-0.012
0.859	114.226	1.677	-0.018
1.023	113.983	1.649	-0.045
0.665	114.514	1.709	0.014
0.650	114.536	1.711	0.016
0.654	114.530	1.710	0.016

A constant elasticity of substitution of approximately 0.72 yields relatively satisfactory estimates of CPI inflation over the examination period. Even with the lowest elasticity estimate in this period, the modified Lloyd-Moulton index outperforms the Lowe index because it allows commodity substitution to a certain degree. Moreover the average elasticity estimates tracking different superlative indices over these years provide fairly close estimates of both index values and inflation rates.

Armknecht and Silver (2012) demonstrated that a simple geometric mean of the weighted arithmetic average and weighted geometric average could approximate the LD index. As an approximate of the LD index, the geometric mean of a Lowe index and a Geometric Young index and the geometric mean of a Young index and a Geometric Lowe index have similar effectiveness in reducing the influence of substitution bias.

Figure 5.3 compares various CPI series under the assumption that the CPI basket is updated every 2 years and used in practice13 months after the weight reference period. Among these indices, the Lowe index is the highest and the Geometric Young index is the lowest, acting as an upper or lower bounds of the other weighted price indices, respectively. Since all the other price indices discussed in section 5.3 follow what are essentially mean index number formulae, they must lie between the two price indices being averaged. For instance, the modified LD index is a weighted arithmetic mean of the Young index and Geometric Young index, and, therefore, must lie between these two indices. The magnitude of the elasticity of substitution (σ) determines which index it approximates more closely. The modified Lloyd-Moulton and the modified LD indices are close approximates to each other, and cannot easily be separately distinguished in the figure. The remaining geometric means of the weighted arithmetic mean and weighted geometric mean can also approximate the modified LD index. Moreover, the values of the geometric mean of the Lowe index and the Geometric Young index, and the geometric mean of the Young index and the Geometric Lowe index are very close to each other. All these alternative indices can be considered improvements upon the Lowe index number formula based on both their mathematical formation, and the empirical results presented in the following figure.

Figure 5.3: Other weighted price indices (January 2002=100)

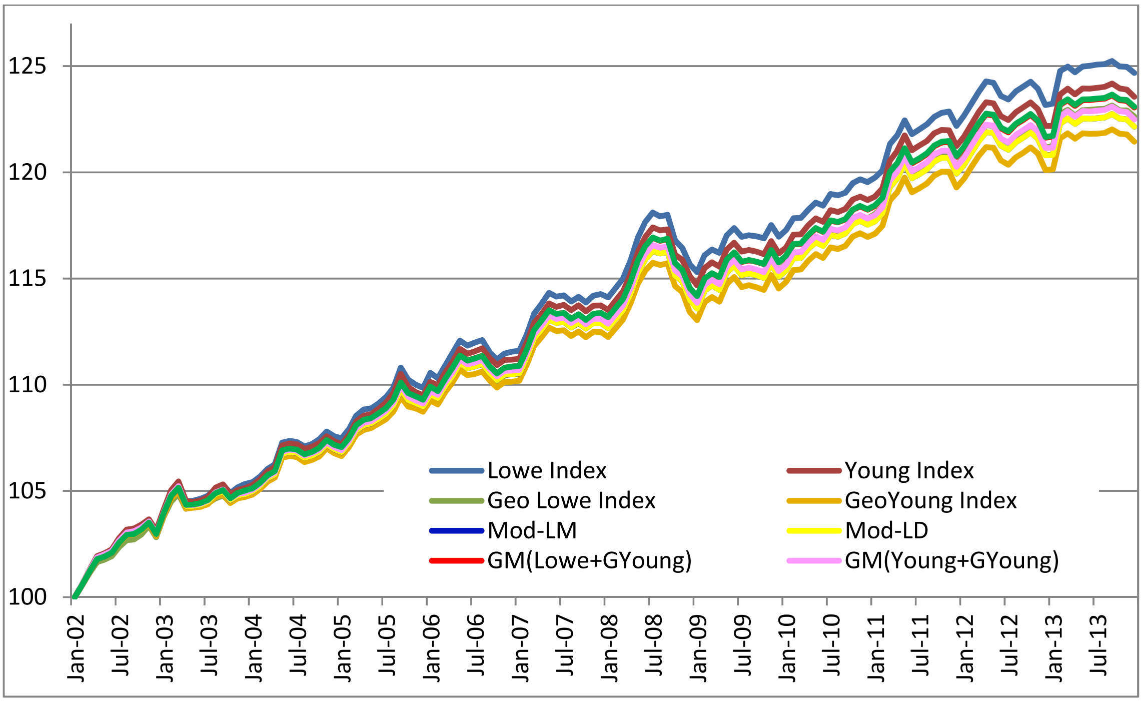


Figure 5.4: Differences in the index values between the Lowe index and the alternative price indices

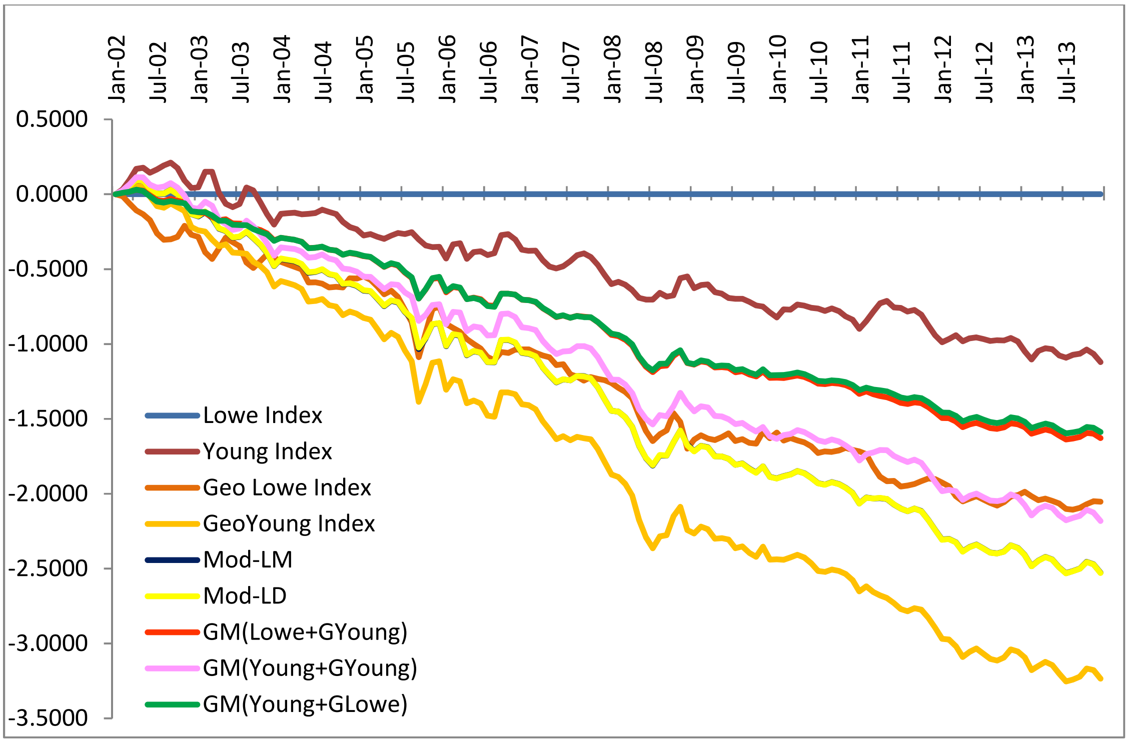
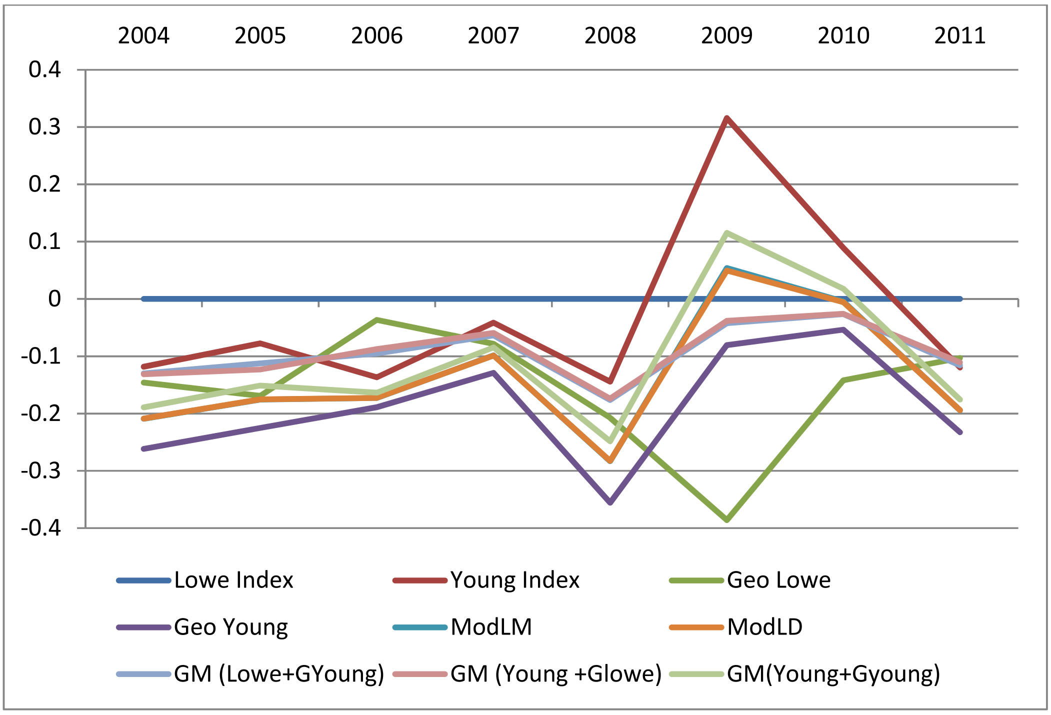


Figure 5.4 gives a clearer picture of how these different CPI series differ from the Lowe index. For most of the period covered by the data, the alternative price indices, compared to the Lowe index, have negative differences in their index values. This indicates that these indices improve upon the performance of the Lowe index to a certain degree, in terms of reducing upper-level substitution bias. Two pairs of price indices—the modified Lloyd-Moulton and modified LD index; and the geometric mean of the Lowe and Geometric Young index and geometric mean of the Young and Geometric Lowe index—are close approximates to each other.

The following figure shows the differences in the annual inflation rate calculated with the Lowe index and those calculated with the other weighted price indices discussed. It can be seen that in some periods the derived annual inflation rates are not always below those calculated with the Lowe index. Specifically, the two modified index series yield higher annual inflation rates in 2009. This is also true for the Young index series and the geometric mean of the Young and Geometric Young series. The positive differences exhibited in the modified Lloyd-Moulton and modified LD index series are due to the inaccuracy of the elasticity estimate. Figure 5.5 also implies that the differences in index values and in annual inflation rates are not always in the same direction.

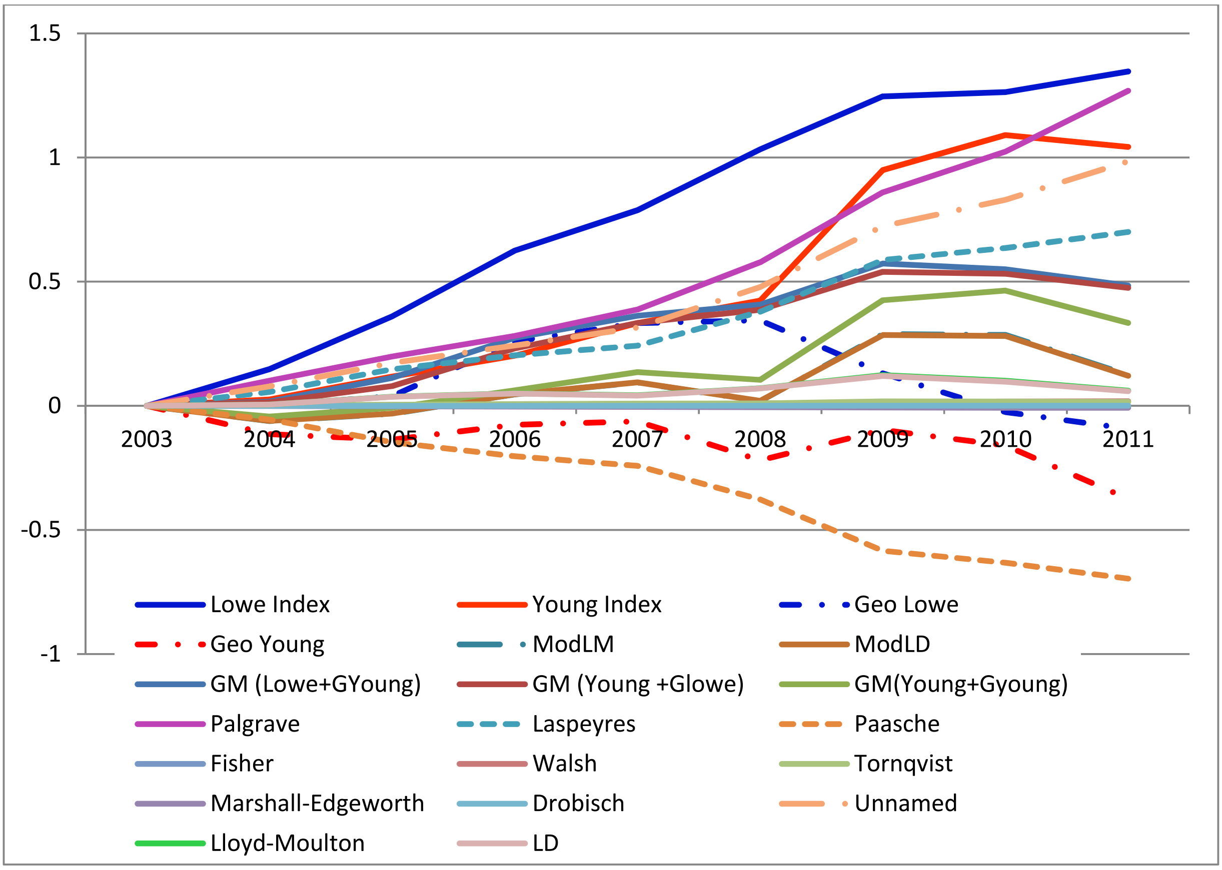
Figure 5.5: Differences in the annual inflation rates between the Lowe index and the alternative price indices



5.4 Comparison with the Target Index

Up to this point, we have examined only how alternate price indices differ from each other and from the Lowe index. To identify the impact that the index number formulae have on the magnitude of substitution bias, it is necessary to examine how all these CPI series differ from the selected theoretical target index—the chained Fisher index in this study. Since we do not have monthly weights, our comparisons with the target index are limited to annual indices. To facilitate the comparison, all the price indices addressed in this paper are shown in the following figure. From this point onwards, for those index formulae which have same data requirement as the Lowe index, the basket is assumed to be updated annually and introduced into the CPI calculation after a 12-month implementation lag. Figure 5.6 shows the differences in the index level between the alternative annual index series and the chained Fisher index:

Figure 5.6: Difference in the index values between the alternative annual index series and the chained Fisher index



Compared with the chained Fisher index, the Lowe index series has the largest positive difference as expected. The three superlative indices and both the Marshall-Edgeworth and Drobisch price indices, which are all symmetrically weighted price indices, closely approximate each other such that they cannot be distinguished in the above figure; whereas, the Un-named price series, which is also symmetrically weighted, yields an observable divergence from the superlative indices. The Lloyd-Moulton index and LD index calculated using a constant elasticity of substitution also approximate the Fisher index quite well over the examination period. Given that the estimated constant elasticity of substitution σ does not suit in different time periods equally well, the divergence between the Lloyd-Moulton index or LD index and the target index vary with the accuracy of the estimated elasticity of substitution. By looking at the difference between the chained Fisher series and the chained Lowe series, chained Young series, chained Palgrave series and chained Laspeyres series, it can be implied that: $P_{ChLo} \geq P_{ChPal} \geq P_{ChL}$. However, the relationships between the chained Young index, and the corresponding chained Laspeyres and chained Palgrave indices are mixed over the sample period. This is determined by the long-run price trends, as well as the elasticity of demand for the products.

Among the price-index series compiled using different index formulae with the same data requirements as the Lowe index, the Geometric Young index has a negative difference compared with the Fisher index. The two geometric means of the arithmetic index and geometric index—the geometric mean of the Lowe index and Geometric Young index, and the geometric mean of the Young index and Geometric Lowe index, diverge similarly from the Fisher index. However, they track the target index less well than the modified Lloyd-Moulton and modified LD indices. To show a quantitative comparison, we present all the index values and associated average annual growth rates in the following table:

Table 5.4: The different chained-CPI series and their geometric annual growth rates

	Indices (2003=100)	**Difference in Index values**	**Annual Growth Rate**	**Difference in Growth Rate**
	2003-2011		(%)	(%)
Fisher	114.389	0.000	1.695	0.000
Walsh	114.408	0.019	1.697	0.002
Törnqvist	114.405	0.016	1.696	0.002
Marshall_Edgeworth	114.381	-0.008	1.694	-0.001
Drobisch	114.390	0.000	1.695	0.000
Un-named	115.374	0.984	1.804	0.109
Laspeyres	115.090	0.700	1.772	0.078
Lloyd-Moulton	114.451	0.061	1.701	0.007
LD	114.448	0.059	1.701	0.007

Palgrave	115.658	1.269	1.835	0.140
Paasche	113.693	-0.696	1.617	-0.078
Lowe	115.736	1.346	1.844	0.149
Young	115.432	1.043	1.810	0.115
Geometric Lowe	114.297	-0.092	1.684	-0.010
Geometric Young	114.015	-0.374	1.653	-0.042
Modified L-M index	114.514	0.125	1.709	0.014
Modified L-D index	114.510	0.121	1.708	0.013
GM (Lowe+ Gyoung)	114.873	0.484	1.748	0.054
GM (Young+ GLowe)	114.865	0.475	1.747	0.053
GM (Young+ Gyoung)	114.723	0.333	1.732	0.037

All the price indices in Table 5.4 are calculated with annually updated baskets. The symmetrically weighted price indices are highlighted in green. As noted in the ILO CPI Manual, the superlative indices closely approximate each other. The other two symmetrically weighted price indices, the Marshall-Edgeworth and Drobisch indices, have the same data requirement as the superlative indices and yield a close approximation to them. The Un-named index, however, diverges significantly from the superlative indices.

The asymmetrically weighted indices require weight information associated with one of the two periods being compared, which are different from the Lowe index. They cannot be used in the CPI in practice due to either the non-revisions policy or the lack of information corresponding to the current price period. Compared with the chained Fisher index, both the Palgrave and Laspeyres indices generate upward bias in the examined period. The Lloyd-Moulton and LD indices, dependent upon the estimation of the elasticity of substitution, are also base-period indices. Allowing product substitution to a certain degree, they reduce the upward substitution bias in the Laspeyres index.

The index series highlighted in purple use the same data required to compile the Lowe index, and therefore can be constructed on a timely basis. This set of price indices is also calculated with annually updated CPI baskets and an implementation lag of 12 months. Because of the existence of persistent long-run price trends in the Canadian economy, the calculated Lowe index is higher than the corresponding Young index for the same period. The geometric Lowe and geometric Young indices are lower than their arithmetic counterparts as expected. The empirical results also indicate that they might be even lower than the superlative indices. The closest approximation to a superlative index in this study as demonstrated by our empirical results is the modified LD index, which is closely approximated by the modified Llyod-Moulton index. This can be shown from very small differences in both index values and growth rates. However, to estimate these two modified indices an estimate of the elasticity of substitution between the commodities included in the index must be available. The three L-D approximates: (i) the geometric mean of the Lowe and Geometric Young indices, and (ii) the geometric mean of the Young and Geometric Lowe indices, and iii) the geometric mean of the Young and Geometric Young indices, dramatically reduce the substitution bias in the Lowe index series and the first two geometric means appear to be quite similar.

6 Conclusion

The choice of index number formula plays an important role in reducing the magnitude of the substitution bias. By choosing an alternative index number formula examined in this paper, even one requiring the same date set that is necessary for estimating the Lowe index, we can still improve upon the performance of the Lowe index to different degrees by reducing the upper-level commodity-substitution bias.

The empirical results suggest that the relationship between the Lowe and the Young indices is uncertain, and depends on whether there are persistent long-term price trends in the economy. The geometric counterparts of these two price indices track the target index more closely than the arithmetic series, and therefore can reduce the substitution bias in the Lowe index. However, they might actually generate downward bias, particularly in the case of the Geometric Young index. Furthermore, it is not easy to justify their application in the official CPI to its users.

This study also demonstrates that the modified Lloyd-Moulton and modified LD indices can be used to form a close approximation to a superlative index, and both closely approximate each other. From the perspective of tracking a superlative index, these two indices could yield satisfactory estimates of the CPI given the availability of a relatively accurate estimator of the elasticity of substitution. The geometric mean of the geometric and arithmetic formulas can also approximate the superlative index quite well, but lack meaningful economic interpretation. Should they be adopted in the practice, the adopting statistical agencies might face unpredictable challenges in the communication with the CPI users. Moreover, the gain of choosing different indices varies with the frequency of basket updates as well as the time lag of implementing CPI baskets.

Considering only the empirical results, we cannot make any recommendation toward changing the index number formula used for upper-level aggregation in the Canadian CPI. To do so, we must fully examine the characteristics of the alternative index number formulae and whether these indices align with the main use of the CPI. However, knowing that these alternative indices can track superlative indices quite well, we can at least use them to obtain a close approximation of the monthly substitution bias in the official CPI.

We are also aware of the limitation of the empirical research. The numerical results presented in this paper are derived from the Canadian data over a certain time period, and as some of these results are time sensitive and might therefore change based on different economic environment, we should, therefore, draw conclusions with caution.

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