

Method for Estimating the Total Consumption Trend Index (CTI Macro)

1. Outline of the Total Consumption Trend Index (CTI macro)

- The Total Consumption Trend Index (CTI macro) is an index to estimate the transition of the total consumption expenditure of all households in Japan (corresponding to the final consumption expenditure of households in the GDP statistics).
- The monthly total consumption expenditures are estimated by means of a time series regression model based on a state space model for every month.
- The Total Consumption Trend Index (CTI macro) is calculated by using the estimated monthly total consumption expenditures. The index is calculated so that the monthly average of the index in the base year (2020) equals 100. In addition to this, as a reference, trend-cycle estimates are calculated.

2. Method of estimation

(1) Estimation of explanatory variables

All of the monthly time series used in estimation of the explanatory variables are decomposed into the trend-cycle component, the seasonal component, the irregular component, and some rapid changes (outliers), respectively. The trend-cycle component and the rapid changes are extracted from each time series to be an explanatory variable.

(2) Calculation of monthly results by means of the time series regression model

The monthly total consumption expenditures are estimated by means of the time series regression model based on the state space model with seasonal adjustment of the final consumption expenditure (in nominal and real term) of households in the GDP statistics as an objective variable and with several consumption related statistics released in every month as well as the Household Consumption Trend Index (Statistics Bureau, Ministry of Internal Affairs and Communications) as explanatory variables. The Total Consumption Trend Index (CTI macro) is calculated by using the estimated monthly total consumption expenditures. The index is calculated so that the monthly average of the index in the base year equals 100^{*1}.

(*1) Please note that the monthly values can be changed depending on the values of the latest month.

(3) Calculation of trend-cycle estimates

As a reference, trend-cycle estimates are calculated by extracting the trend and the cycle components from the total consumption trend index obtained in (2) above.

3. Details of the method of estimation

(1) Estimation of explanatory variables

- For estimation, we make the well-known assumption in the area of seasonal adjustment that each monthly time series used in estimation of the explanatory variables (Y_n) consists of trend (T_n), cycle (C_n), seasonal (S_n), outliers (D_n^k), and irregular (I_n) components^{*2}.

- These components are estimated from the linear Gaussian state space model^{*3} below.

[Observation equation]

$$Y_n = T_n + C_n + S_n + \sum_{k=1}^K D_n^k + I_n \quad I_n \sim N(0, \sigma_I^2)$$

[State equation]

$$T_n = 2T_{n-1} - T_{n-2} + \varepsilon_n^T \quad \varepsilon_n^T \sim N(0, \sigma_T^2)$$

$$C_n = \alpha_1 C_{n-1} + \alpha_2 C_{n-2} + \varepsilon_n^C \quad \varepsilon_n^C \sim N(0, \sigma_C^2)$$

$$S_n = -\sum_{i=1}^{11} S_{n-i} + \varepsilon_n^S \quad \varepsilon_n^S \sim N(0, \sigma_S^2)$$

$$D_n^k = \beta^k Z_n^k \quad (k = 1, 2, \dots, K) \quad (Z_n^k: \text{Variables specifying the type of dummy})$$

(*2) Variables to adjust calendar factors such as leap-year effects are not prepared.

(*3) The state space model is a model consisting of the “observation equation” and the “state equation.” Based on the assumption that there exists an unobservable state behind the observed value, the state values are estimated from the observed values. For the frequently used linear Gaussian state space model, there is an effective algorithm to estimate the state values called the Kalman Filter.

- For each monthly time series, the trend (T_n), the cycle (C_n) and the outliers (D_n^k) components estimated from the above formulae are extracted to be the explanatory variable (X_n) used in (2) below.

$$X_n = T_n + C_n + \sum_{k=1}^K D_n^k : \text{Explanatory variables used in (2) below}$$

(2) Calculation of monthly results by means of the time series regression model

- The monthly total consumption expenditure (y_n) is estimated by the time series regression model based on the state space model below. The model considers the change of the relation among the explanatory variables by introducing the time-varying regression coefficients (β_n^i), which are changed dynamically (Time-varying linear regression model). In addition, the second equation of the Observation equation expresses the restriction on the monthly estimated value (y_n) to be consistent with the quarterly observation value (Y_n^{GDP}).

[Observation equation]

$$y_n = T_n + \sum_{i=1}^m \beta_n^i X_n^i$$

$$Y_n^{GDP} = y_n + y_{n-1} + y_{n-2} + \varepsilon_n^{GDP} \quad \varepsilon_n^{GDP} \sim N(0, \sigma_{\varepsilon^{GDP}}^2) \quad (i = 1, 2, \dots, m)$$

[State equation]

$$T_n = 2T_{n-1} - T_{n-2} + \varepsilon_n^T \quad \varepsilon_n^T \sim N(0, \sigma_T^2)$$

$$\beta_n^i = \beta_{n-1}^i + \varepsilon_n^{\beta^i} \quad (i = 1, 2, \dots, m) \quad \varepsilon_n^{\beta^i} \sim N(0, \sigma_{\beta^i}^2)$$

- The total consumption trend index (CTI_n^{Total}) is calculated by multiplying the ratio of the total consumption expenditure of the current month (y_n) by the monthly average of the total consumption expenditure in the base year (y_{base}) by 100. Note that the total consumption expenditures in the base year can be changed in each estimation, because the monthly total consumption expenditures are re-estimated backward in every time.

$$CTI_n^{Total} = \frac{y_n}{y_{base}} \times 100$$

(3) Calculation of trend-cycle estimates

- Trend-cycle estimates are calculated from the total consumption trend index (CTI_n^{Total}) to delete the seasonal and irregular components by means of the Census Method (X-12-ARIMA).
- The option of X-12-ARIMA is used for a calculation similar to that used in the calculation of trend-cycle estimates of the total consumption expenditure for the total households in the Household Consumption Trend Index (CTI micro).

4. Explanatory variables and dummy variables used for the estimation

As for the explanatory variables used for the estimation, dummy variables, etc., those optimums are selected based on AIC.

(1) Explanatory variables used for the estimation

a. Total Consumption Trend Index: nominal term

- Objective variable
 - Quarterly Estimates of GDP, nominal seasonal adjustment result, final consumption expenditure of households (Cabinet Office)
- Explanatory variables
 - Household Consumption Trend Index (CTI micro), original series (nominal term) for total households, consumption expenditure (Statistics Bureau, Ministry of Internal Affairs and Communications)
 - Preliminary Report on the Current Survey of Commerce, original value, retail (sales value) (Ministry of Economy, Trade and Industry)
 - Monthly Survey on Service Industries, service industry (sales) (Statistics Bureau, Ministry of Internal Affairs and Communications)*4

(*4) As for the data before the start of the survey (October 2008), a prediction value based on the time series regression model using the results of the Indices of Tertiary Industry Activity as the explanatory variable is used.

b. Total Consumption Trend Index: real term

- Objective variable
 - Quarterly Estimates of GDP, real seasonal adjustment system, domestic final consumption expenditure of households (Cabinet Office)
- Explanatory variables
 - Household Consumption Trend Index (CTI micro), original series (real term) for total households, consumption expenditure (Statistics Bureau, Ministry of Internal Affairs and Communications)
 - Indices of Tertiary Industry Activity, original index, broad-ranging personal service (Ministry of Economy, Trade and Industry)
 - Indices of industrial production, original index, consumer goods (Ministry of Economy, Trade and

Industry)

(2) Dummy variables for each explanatory variable*⁵

a. Total Consumption Trend Index: nominal term

- Household Consumption Trend Index (CTI micro) (Statistics Bureau, Ministry of Internal Affairs and Communications)
March 2011: AO*⁶, April 2011: AO, March 2014: AO, April 2014: AO, May 2014: AO,
September 2019: AO, October 2019: LS*⁶, February - April 2020: Rp*⁶, May - October 2020: Rp,
June 2020: AO, January 2021: AO, February 2021: AO, May 2021: AO, August 2021: AO
- Preliminary Report on the Current Survey of Commerce (Ministry of Economy, Trade and Industry)
March 2011: AO, April 2011: AO, May 2011: AO, March 2014: AO, April 2014: AO,
September 2019: AO, October 2019: AO, February - April 2020: Rp, June 2020: LS, January 2021: AO,
April - May 2021: TL*⁶, August 2021: AO
- Monthly Survey on Service Industries (Statistics Bureau, Ministry of Internal Affairs and Communications)
February - April 2020: Rp, June 2020: LS, August - October 2020: Rp, January 2021: LS,
August 2021: AO

b. Total Consumption Trend Index: real term

- Household Consumption Trend Index (CTI micro) (Statistics Bureau, Ministry of Internal Affairs and Communications)
March 2011: AO, April 2011: AO, March 2014: AO, April 2014: AO, May 2014: AO,
September 2019: AO, October 2019: LS, February - April 2020: Rp, May - October 2020: Rp,
June 2020: AO, January 2021: AO, February 2021: AO, May 2021: AO, August 2021: AO
- Indices of Tertiary Industry Activity (Ministry of Economy, Trade and Industry)
March 2011: AO, April 2011: AO, May 2011: AO, March 2014: AO, April 2014: LS,
September 2019: AO, October 2019: AO, February - April 2020: Rp, June 2020: LS,
July - October 2020: Rp, January 2021: AO, May 2021: AO, August 2021: AO
- Indices of industrial production (Ministry of Economy, Trade and Industry)
March 2011: AO, April 2011: AO, May 2011: AO, March - May 2020: Rp, July 2020: LS, May 2021: AO,
August 2021: AO, September 2021: AO, October 2021: AO

(*⁵) Dummy variables already set forth are revised as needed, considering future transition of data.

(*⁶) AO: additive outlier, LS: level shift, Rp: ramp, TL: temporally level shift

References

Takabe Isao. (2018), The Consumption Trend Index (CTI): Estimation Methods for the Macro Consumption Trend ,
Research Memoir of Official Statistics, No.75, Statistical Research and Training Institute Ministry of Internal
Affairs and Communications Japan, 21-40 (in Japanese).

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<https://www.stat.go.jp/info/kenkyu/skenkyu/index.html>