Appendix 4 Calculation of the CPI by Laspeyres' Chain Index method

The weighted arithmetic mean with a fixed basket in the base period preceding the comparison period (Laspeyres formula) is used in many countries including Japan to calculate indices by combining the ratio of prices (indices) in the base period and the comparison period by using the weight of the base period. The Laspeyres formula is subdivided into the "fixed-base method" in which the index is calculated with the rate of consumption expenditure in the base year as the weight, and the "chain-linking method" in which the index of the current year, calculated based on the rate of consumption expenditure in the previous year as the weight, is multiplied every year.

Indices published in Japan are calculated by the fixed-base method, while those computed by the chain-linking method are also published as reference indices.

1 Basic formulas

In the chain-linking method, an index of a period is calculated referred to the period immediately before that period (known as the "link index"), and the indices of two consecutive periods are multiplied in series to find the index (known as the "chain index").

The formulas used in this method are shown in 1) to 3). A link is made once a year, and the weight is revised every year using yearly average of the FIES (for households composed of two or more persons) in the previous year. The Laspeyres formula is used for finding the link indices.

1) Group indices excluding fresh food (e.g., "All items, less fresh food") (monthly indices)

The price index ratio by item used for the Laspeyres link index is calculated by dividing the price index of an item in the comparison period by the price index of that item[51] in December the previous year.[52]

[51] The "Indices of items for Japan," calculated by the fixed-base method for each base, is used for price indices by item which are used for calculating the price index ratios by item.

[52] Comparison of the chain-linking and fixed-based methods for each month of the year following the base year (2016) shows that the weight reference year is the same in both methods (weight are refer to 2015), but the resulting index is not always the same because of differences in calculation such as the use of the price index ratio in December of the previous year for the price index ratio by item in the chain-linking method and other reasons.

<Group indices excluding fresh food (monthly indices)>

(Laspeyres link index (*L*)) $I_{y,m}^{(L)} = \dfrac{\sum_{i=1}^{n} \dfrac{I_{y,m,i}}{I_{y-1,12,i}} w_{y-1,i}}{\sum_{i=1}^{n} w_{y-1,i}}$

(Laspeyres chain index(*C*)) $I_{y,m}^{(C)} = I_{0,12} \times \prod_{Y=1}^{y-1} I_{Y,12}^{(L)} \times I_{y,m}^{(L)}$

Where, $I_{0,12}$ is the Laspeyres chain index in December 2015 with 2015 = 100[53]

(*Y*, *y*: Year, *m*: Month, 0: Base year, *i*: Item, *n*: No. of items, *w*: Weight)

2) "Fresh fish & seafood," "Fresh vegetables" and "Fresh fruits" (monthly indices)

The price index ratios by item used for the Laspeyres link index are given by dividing the price index of an item in the comparison period by the annual average price index of that item in the previous year.[54]

<"Fresh fish & seafood," "Fresh vegetables" and "Fresh fruits" (monthly indices)>

(Laspeyres link index (*L*)) $I_{y,m}^{(L)} = \dfrac{\sum_{i=1}^{n} \dfrac{I_{y,m,i}}{I_{y-1,i}} w_{y-1,m,i}}{\sum_{i=1}^{n} w_{y-1,m,i}}$

(Laspeyres chain index (*C*)) $I_{y,m}^{(C)} = I_0 \times \prod_{Y=1}^{y-1} I_Y^{(L)} \times I_{y,m}^{(L)}$, where $I_0 = 100$

(*Y*, *y*: Year, *m*: Month, 0: Base year, *i*: Item, *n*: No. of items, *w*: Weight)

3) Upper level group indices including fresh food (e.g., "All items" and "Food") (monthly indices)

Using the above formula showed in 1), the upper level group indices are calculated from the group indices excluding fresh food calculated in 1), and "Fresh fish & seafood," "Fresh vegetables" and "Fresh fruits" calculated in 2).

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[53] For each month of 2015, the Laspeyres link index, with December 2014 = 100, is derived first with the 2010-base items and weight (2014 yearly averages). Then, Laspeyres link index in December 2015 is divided by the average of January to December 2015 to find the Laspeyres chain index for December 2015 ($I_{0,12}$) with 2015 = 100.

[54] The annual average price index of fresh food is derived by averaging the monthly price index with the monthly weight (refer to "III Chapter 5, 4 (1) Average indices for calendar year").

The monthly index is published in time with the publication of confirmed figures for the month in question, but for confirmed figures for January, a temporary index is calculated by using the weight two years ago due to incompletion of the previous year's weight. When the results of the FIES are published, a confirmed index is calculated with the previous year's weight and a revised index extending back to January is published in time with the publication of confirmed figures for February.

(Example of monthly index calculation)

To use a simple example, it is assumed that there is a subgroup "Meats" consisting of two items ("Beef" and "Pork"), and the price index for "beef" rises every year, and the price index for "pork" does not change. The weight of "beef" shrinks and the weight of "pork" expands as "beef" price rises.

Year and month	Price index*		Year	Weight	
	Beef	Pork		Beef	Pork
Dec., 00	100	100	00	10	10
Dec., 01	200	100	01	8	12
Jun., 02	400	100			

* Average in year 00 = 100

The index for "Meats" in June 02 calculated by the fixed-base method is given by averaging the price indices of "beef" and "pork" in June 02 with the weights of 00 as follows:

$$\frac{400 \times 10 + 100 \times 10}{10 + 10} = 250$$

To find the Laspeyres chain index for June 02, the Laspeyres link index for December of each year till the previous year (indices with the index for December of the previous year as 100) and Laspeyres link index for June 02 are calculated first.

The Laspeyres link index for December 01 is given by averaging the price index ratio of December 01 and December 00 with the weight of 00:

$$\frac{\frac{200}{100} \times 10 + \frac{100}{100} \times 10}{10 + 10} = 1.5$$

The Laspeyres link index of June 02 is given by averaging the price index ratio of June 02 and December 01 with the weight of 01:

$$\frac{\frac{400}{200} \times 8 + \frac{100}{100} \times 12}{8 + 12} = 1.4$$

Finally, the Laspeyres chain index of June 02 is calculated by multiplying the Laspeyres link indices of December 00 (100 is assumed here), December 01 and June 02:

$$100 \times 1.5 \times 1.4 = 210$$

2 Calculation of weights

(1)Weight reference period

The weight of each item for the Laspeyres link index is calculated from the annual average monthly expenditure by item per household in the previous year of the comparison period for that item, mainly obtained from the FIES for households consisting of two or more persons.

(2)Allocation rates

In calculating weights used for the fixed-base method, when more than one index item corresponds to one FIES item, the allocation rate is determined by the expenditure ratio which is obtained from the special tabulation of the of FIES and other statistics. In calculating weights for the Laspeyres link index, the allocation rates of the base year (refer to "III Chapter 4, 2 Calculation of the weights for the basic classification indices") are used until the next base revision due to restrictions in estimation data, but the allocation rates for part of items for which annual estimation data is available is revised every year.

(3) Weights for "Pocket money," etc.

In calculating weights used for the fixed-base method, the weights of "Pocket money" and "Social expenses" in the FIES are allocated based on the result of the NSFIE and their weights are calculated accordingly. As the data is obtained only once in five years, the allocation rate in calculating the weights for the Laspeyres link index is fixed until the next base revision.

(4) Weights for the imputed rent

In calculating weights used for the fixed-base method, the weights for "Imputed rent" are calculated for the base year according to the result of the NSFIE. The weights for the Laspeyres link index are given by multiplying the weight for "Imputed rent" in the base year by the increase-decrease rate of the price index of "Imputed rent" (for each municipality and other classifications) from the base year to the reference period.

(5)Monthly weights for fresh food

In calculating weights used for the fixed-base method, the monthly weights for fresh food are determined by the ratio of the monthly purchase quantity to the annual average purchase quantity by item in the FIES. The annual weights for the Laspeyres link index are revised every year according to (1), while the ratio of the monthly purchase quantity for calculating the monthly weights is fixed until the next base revision.[55]

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[55] The ratio of monthly purchase quantity could be revised every year, but this causes irregular fluctuation which is to be included in the weights, and may tend to cause the drift phenomenon in the chain indices. In calculating monthly weights for link indices, the calculation and adjustments in b) and c) of (3) of "III Chapter 4 2 Calculation of the weights for the basic classification indices" need to be repeated until the consistency is achieved between annual and monthly weights for items, and between monthly weights for groups and those for items. It brings the different monthly weights for both methods in the base years.

3 Calculation of annual average index

The annual average index is the simple arithmetic average of monthly indices from January to December calculated by groups as described in Section 1.[56]

4 Calculation of the rate of change

The same formula used in the fixed-base method is used. The indices containing fractions are used for calculation, but the figures in the statistical tables are rounded off to the first decimal place.

5 Calculation of contribution

The contributions to month-to-month and year-ago month ratios of the all items index are given by the formulas shown below. The indices containing fractions are used for calculation, but the figures in the statistical tables are rounded off to the second decimal place.

(1) Contribution to month-to-month ratio

1) Items excluding fresh food

$$\text{Contribution of item } i \text{ (February to December)} = \left[\frac{1}{I^{(L)n}_{y,m-1}} \cdot \left(\frac{I_{y,m,i} - I_{y,m-1,i}}{I_{y-1,12,i}}\right) \cdot \frac{w_{y-1,i}}{\sum_{i=1}^{n} w_{y-1,i}}\right] \times 100$$

$$\text{Contribution of item } i \text{ (January)} = \left[\left(\frac{I_{y,1,i} - I_{y-1,12,i}}{I_{y-1,12,i}}\right) \cdot \frac{w_{y-1,i}}{\sum_{i=1}^{n} w_{y-1,i}}\right] \times 100$$

2) Fresh food

$$\text{Contribution of item } i \text{ (February to December)} = \left[\frac{1}{I^{(L)n}_{y,m-1}} \cdot \frac{I^{(L)j}_{y-1}}{I^{(L)j}_{y-1,12}} \cdot \left(\frac{I_{y,m,i} \cdot w_{y-1,m,i} - I_{y,m-1,i} \cdot w_{y-1,m-1,i}}{I_{y-1,i} \cdot \sum_{i=1}^{n} w_{y-1,i}}\right)\right] \times 100$$

$$\text{Contribution of item } i \text{ (January)} = \left[\left(\frac{I^{(L)j}_{y-1}}{I^{(L)j}_{y-1,12}} \cdot \frac{I_{y,1,i}}{I_{y-1,i}} - 1\right) \cdot \frac{w_{y-1,1,i}}{\sum_{i=1}^{n} w_{y-1,i}}\right] \times 100$$

Contribution: Contribution of item i to month − to − month ratio of all items index in month m of year y

$I_{y,(m,)i}$: Price index of item i in year y (month m)

$w_{y,(m,)i}$: Weight of item i in year y (month m), j: Group of fresh food, n: All items

$I^{(L)x}_{y,m}$: Laspeyres link index in year y (month m) in the range of target in x

Note that this calculation sometimes brings inconsistency between month-to-month ratios of items and contributions.

(2) Contribution to year-ago month ratio

1) Items excluding fresh food

$$\text{Contribution of item } i = \left[\frac{1}{I^{(L)n}_{y-1,m}} \cdot \left(\frac{I_{y-1,12,i} - I_{y-1,m,i}}{I_{y-2,12,i}}\right) \cdot \frac{w_{y-2,i}}{\sum_{i=1}^{n} w_{y-2,i}} + \frac{I^{(L)n}_{y-1,12}}{I^{(L)n}_{y-1,m}} \cdot \left(\frac{I_{y,m,i}}{I_{y-1,12,i}} - 1\right) \cdot \frac{w_{y-1,i}}{\sum_{i=1}^{n} w_{y-1,i}}\right] \times 100$$

[56] The annual average indices for the chain-linking method were calculated by linking average prices in the previous year in 2010-base, while simple arithmetic averages of monthly indices are used in 2015-base.

2) Fresh food

$$\text{Contribution of item } i = \left[\frac{1}{I^{(L)n}_{y-1,m}} \cdot \frac{I^{(L)j}_{y-2}}{I^{(L)j}_{y-2,12}} \cdot \left(\frac{I_{y-1,12,i} \cdot w_{y-2,12,i} - I_{y-1,m,i} \cdot w_{y-2,m,i}}{I_{y-2,i} \cdot \sum_{i=1}^{n} w_{y-2,i}}\right) + \frac{I^{(L)n}_{y-1,12}}{I^{(L)n}_{y-1,m}} \cdot \left(\frac{I^{(L)j}_{y-1}}{I^{(L)j}_{y-1,12}} \cdot \frac{I_{y,m,i}}{I_{y-1,i}} - 1\right) \cdot \frac{w_{y-1,m,i}}{\sum_{i=1}^{n} w_{y-1,i}}\right] \times 100$$

Contribution: Contribution of item i to year − ago month ratio of all items index in month m of year y

$I_{y,(m,)i}$: Price index of item i in year y (month m)

$w_{y,(m,)i}$: Weight of item i in year y (month m), j: Group of fresh food, n: All items

$I^{(L)x}_{y,m}$: Laspeyres link index in year y (month m) in the range of target in x

Note that this calculation sometimes brings inconsistency between year-ago month ratios of items and contributions.

6 Index grouping

Monthly and annual average indices are calculated for the basic classification indices for Japan.

The Laspeyres chain indices obtained in the former bases are recalculated since 1986 indices by the 2015-base formula. Weights used for obtaining the Laspeyres chain indices released in each of the former bases are used as they are[57] to obtain the Laspeyres link indices for the relevant years. The same procedure to link indices within a base period is used for linking indices of different base periods. For example, the monthly Laspeyres chain index in 2011 are obtained by multiplying the Laspeyres chain index for December 2010 calculated with 2005-base items and weights by the relevant monthly Laspeyres link index in 2011 calculated with 2010-base items and weights.

[Reference 1] History of index calculation using the chain-linking method

The indices published in Japan have been calculated using the fixed-base method since August 1949 to measure price fluctuations in a fixed consumption patterns. The indices obtained from the chain-linking method, which reflect annual changes in the consumption patterns, have been released as the reference annual average indices since 1975-base, and reference monthly indices for all items excluding fresh food have also been released from 2005-base.

In 2015-base, chain-linked monthly indices for all items including fresh food are released together with the chain-linking contribution of groups and items.

The indices obtained from the Laspeyres chain-linking method are released since 1986 indices with the publication of indices for January 2017. These recalculated indices (rate of change) are different from that of the former bases used to be published due to the difference of calculation formulas etc.

[Reference 2] Characteristics of chain-linking method

Generally speaking, “the effect of weights”, “effect of reset” and “drift phenomenon” are typical differences between the chain-linking method and fixed-base method.

[57] Monthly weights for fresh food are newly calculated from annual averages of weights for linking of individual bases and monthly weights of the base year.

With “the effect of weight”, in addition to price fluctuation, the change in weights is reflected in the index and the rate of change in the chain-linking method, in which the weight is revised every year, if there is an item the rate of consumption of which increases or decreases with the fall or rise of price. However, the relations between price fluctuations and changes in the rate of consumption are not uniform, and the direction and significance largely differ depending on the characteristics of the item. It should, therefore, be noted that differences from the indices in the fixed-base method are not always seen in either the upper or lower parts.

“The effect of reset” causes differences from the fixed-base method in the contribution (degree of effect) of items because the indices level of items are reset every year in the chain-linking method, in which the index value of every item is calculated by assuming the index = 100 in December of previous year. If the price of an item has largely dropped, and the price decline continues after the index value calculated by the fixed-base method has become considerably small, the range of drop would be greater in the chain-linking method than in the fixed-base method.

In the chain-linking method, the following phenomenon is pointed out: If the price of an item repeatedly rises and falls, the index values of the upper level group do not return to the normal level even if the item in question returns to the original price level. This is so called the “drift phenomenon.”

[Reference 3] “Link of price in December of previous year” and “link of average price of previous year”

In the chain-linking method, the chain index is calculated by multiplying the link indices of two consecutive periods in series, and there are two types of periods to link (linking points). These are, “link of the price in December of previous year” and “link of the average price of previous year.” As shown in Section 1 Basic formulas, the “link of the price in December of previous year” is used for items excluding fresh food, and “link of the average price of previous year” is used for fresh food due to the following reasons:

While a “drift phenomenon” takes place in some cases in the chain-linking method, this phenomenon is seen more frequently in the “link of the price in December of previous year” than in the “link of the average price of previous year.”

At the same time, the “link of the average price of previous year” sometimes causes the indices of the upper level group to change due to the change of the linking point in December and January of the next year, even if the price of the item in question does not change (this is called the occurrence of “discontinuation.”).

The “discontinuation” is considered undesirable in general, and for this reason, the “link of the price in December of previous year” is used for the items other than fresh food. The fresh food, however, contains seasonal items whose price repeatedly rises and falls in one year cycle, causing a large-scale “drift phenomenon” when the “link of the price in December of previous year” is used, resulting in the indices of the upper level group to remain in the upper level even though the indices of the items in the same category return to the original level. That is the reason for using the “link of average price of previous year” exceptionally to fresh food.

[Reference 4] Basics of decomposition of contribution for the chain-linking method[58]

(1) Decomposition of the contribution for the year-ago month ratio in fixed-base indices

The contribution is decomposed by decomposing the year-ago month formula for the all items index into the sum of rates of change for item i. When the year-ago month ratio of the fixed-base index is decomposed:

Year-ago month of all items index =

$$\frac{I_{y,m} - I_{y-1,m}}{I_{y-1,m}} \times 100 = \frac{1}{I_{y-1,m} \cdot \sum_{i=1}^{n} w_{y=0,i}} \left(\sum_{i=1}^{n} \frac{I_{y,m,i} - I_{y-1,m,i}}{I_{y=0,i}} \cdot w_{y=0,i} \times 100 \right) \times 100$$

In the above formula, the term in the parentheses represents the sum of rates of change for item *i*. By removing Σ from this term, it represents the contribution of item *i*.

$$\text{Contribution of item } i = \left(\frac{1}{I_{y-1,m} \cdot \sum_{i=1}^{n} w_{y=0,i}} \cdot \frac{I_{y,m,i} - I_{y-1,m,i}}{I_{y=0,i}} \cdot w_{y=0,i} \times 100 \right) \times 100$$

Because $I_{y=0,i} = 100$,

$$\text{Contribution of item } i = \left(\frac{I_{y,m,i} - I_{y-1,m,i}}{I_{y-1,m}} \cdot \frac{w_{y=0,i}}{\sum_{i=1}^{n} w_{y=0,i}} \right) \times 100$$

This indicates the consistency with the formula in "III Chapter 5, 6 Calculation of the contribution."

(2) Decomposition of the contribution for the year-ago month ratio in chain-linked indices (Series groupings excluding fresh food)

Similar to the technique used for the fixed-base method, the formula for the year-ago month ratio of the all items index for the chain-linked method is decomposed to the sum of rates of change for item i.

Considering the year-ago month ratio in the period from month *m* of year *y* - 1 to month *m* of year *y*, with linking point December of *y* - 1 as the boundary, the link index in the period from month *m* of year *y* - 1 to December of *y* - 1 is based on the weight of *y* - 2, while the link index in the period from December of year *y* - 1 to month *m* of year *y* is based on the weight of *y* - 1. Hence, the calculation of decomposition is separated to the contribution from month *m* of year *y* - 1 to December of year *y* - 1, and the contribution from December of year *y* - 1 to month *m* of year *y*. Of the contribution formula in 5 (2) 1) "Items excluding fresh food," the first term: $\frac{1}{I_{y-1,m}^{(L)n}} \cdot \left(\frac{I_{y-1,12,i} - I_{y-1,m,i}}{I_{y-2,12,i}} \right) \cdot \frac{w_{y-2,i}}{\sum_{i=1}^{n} w_{y-2,i}}$ corresponds to the contribution from month *m* of year *y* - 1 to December of year *y* - 1, and the second term: $\frac{I_{y-1,12}^{(L)n}}{I_{y-1,m}^{(L)n}} \cdot \left(\frac{I_{y,m,i}}{I_{y-1,12,i}} - 1 \right) \cdot \frac{w_{y-1,i}}{\sum_{i=1}^{n} w_{y-1,i}}$ corresponds to the contribution from December of year *y* - 1 to month *m* of year *y*.

Note that this calculation rarely brings inconsistency between year-ago month ratios of items and contributions when signs between the first term and the second term are different.

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[58] For the derivation of the formula for decomposing the contribution relating to the year-ago month ratio, refer to Exhibit 11 of "2015-Base Consumer Price Index Revision Plan (Draft) - Attached Material" (http://www.stat.go.jp/info/guide/public/cpi/pdf/150717_a3.pdf) (available only in Japanese).